Optimization under Uncertainty: Exact Algorithms and Approximation Schemes

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Abstract

A large number of problems in production planning and scheduling, location, transportation, finance, and engineering design require that decisions be made in the presence of uncertainty. Uncertainty, for instance, governs the prices of fuels, the availability of electricity, and the demand for chemicals. Stochastic optimization is the branch of applied mathematics that provides systematic tools to prudent decision-making under uncertainty.

A key difficulty in stochastic optimization is in dealing with an uncertainty space that is huge and which leads to very large-scale optimization models. Decision-making under uncertainty is often further complicated by the presence of integer decision variables to model logical and other discrete decisions in a multi-period or multi-stage setting.

This paper reviews methodology, algorithms, and software that have been developed to cope with the complexity of optimization problems under uncertainty. We discuss and contrast the classical two- and multi-stage stochastic programming, probabilistic (chance-constraint) programming, and fuzzy programming. The advantages and shortcomings of these models are reviewed and illustrated through examples. Applications in process systems engineering receive special attention in this review.

Finally, we propose several main areas for future development in this field. These include development of polynomial-time approximation schemes for multi-stage stochastic programs and the application of global optimization algorithms to multi-stage and chance-constraint formulations.