

# OPTIMAL DESIGN AND PLANNING OF SUPPLY CHAINS WITH INTEGRATED FORWARD AND REVERSE FLOWS UNDER UNCERTAINTY

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## *Abstract*

Markets increasing competition coupled with a growing concern with the environment have created a new way of thinking when designing and planning supply chains. The aim of increasing supply chains' sustainability has been emerging. One step towards the achievement of such aim is the integration of reverse logistics into the traditional supply chains. This has created the closed-loop supply chains where not only the supply of final customers' is considered but also reverse logistics aspects are well thought-out. The complexity of the supply chains has therefore increased and the associated decisions are even more subject to uncertainties calling for the need of developing efficient tools that can help decision making of such systems.

In this paper, a mixed integer linear programming (MILP) formulation is developed for the design and planning of supply chains, considering simultaneously production, distribution and reverse logistics activities, taking into account products' demand uncertainty with the goal of maximizing the expected net present value. The results provide details on sizing and location of plants, warehouses and retailers, definition of processes to install, forward and reverse flows and inventory levels. The model is applied to a representative European supply chain case study in order to show its applicability.

## *Keywords*

Closed-loop Supply Chain, Design and Planning, Uncertainty, MILP.

## **Introduction**

In recent years, there has been a growing concern with the environment, particularly in what is related to energy consumption and natural resources limitation that may lead to resources scarcity. Society concerns and legislation have been forcing companies to consider the re-usage of their non-conform or end-of-life products. To deal with this new flow, supply chains need to be reconfigured so as to

simultaneously deal with forward and reverse and the design and planning decisions need to be considered simultaneously with reverse logistics activities (Barbosa-Póvoa et al., 2007; Seuring and Muller, 2008). For this purpose, the supply chain should now be seen as a closed loop system (Guide and Van Wassenhove, 2002; Papageorgiou, 2009) where reverse logistics activities are

included encompassing the transportation and reprocessing of collected products. In their review, Melo et al. (2009) concluded that despite its importance, few works have addressed the design of supply chains with reverse flows. The same conclusion is corroborated by Salema et al. (2010) who stated that the number of published works where both forward and reverse flows are taken into account simultaneously is less than the ones that treat them separately. In particular, the treatment of such systems under an uncertainty environment is still in its infancy. Some works have appeared dealing with uncertainty in forward supply chain (Tsiakis et al., 2001, Guillén et al., 2005; Georgiadis et al., 2011) but the treatment of supply chains with forward and reverse flows has not yet been treated adequately.

In this work we propose a mathematical model for the problem of designing and planning of supply chains integrating simultaneously forward and reverse flows, in which products' demand is uncertain and represented through a scenario tree. The developed model is applied to an European supply chain case study, where several network structures are analyzed, ranging from simple forward to closed-loop supply chains, where markets can be supplied directly from plants, warehouses or through the retailers.

The paper is structured along the following sections: problem definition, model formulation, model implementation and conclusions.

### Problem Definition

The problem addressed in this work has the objective of determining the supply chain structure along with planning decisions that maximizes the expected net present value (NPV), taking into account the uncertainty in products' demand. Uncertainty is captured in terms of a number of scenarios that are possible to happen during the entire planning time horizon having different associated probabilities. The resulting supply chain structure is capable to deal with all considered scenarios. The demand is known for the first period and the scenarios are built as shown in figure 1.

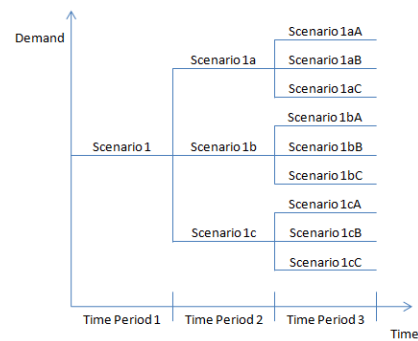


Figure 1. Scenarios construction

In this approach the information regarding products' demand in a given time period becomes available at the end of the preceding period and this result in each scenario branch breaking into multiple branches at these points. Each three branches with origin in a scenario from a previous time period, try to represent optimistic, most likely and pessimistic situations, having each one an expected probability of occurrence.

The present supply chain is formed by four echelons: plants with a set of available processes to be installed; warehouses where final products are assembled and stored; retailers which store the final products and deliver them to markets. It is also considered that plants can exchange all type of products among them, including raw materials or intermediate products. In order to generalize the supply chain and to be able to adapt it to several situations of the real world, it is also analyzed the case where plants and warehouses can directly deliver products to markets. In the reverse flow, products are collected by the retailers from nearby markets and then are sorted. Products that are too damaged are sent to disposal while products in their end of life are directed to plants to be disassembled and reprocessed. Non-conforming products are sent to warehouses to be repacked. In figure 2 it is possible to see a generic representation of this network. Each echelon can have multiple entities, generating all potential flow combinations.

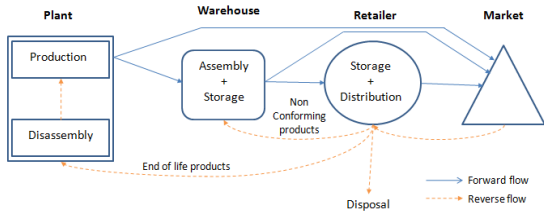


Figure 2. Network superstructure

Having such network the problem can be generally described as follows:

**given** the input data: a) upper and lower bounds for the infrastructures capacities expansions, b) inventory costs at the warehouses and retailers, c) initial capacity and initial inventory level in the existing infrastructures, d) price of each final product, e) estimation of products' demand in terms of scenarios, f) travel distance between each pair of entities of the network, g) investment costs, operational costs and unitary transportation costs, h) price of each raw material, i) percentage of collected products that are sent to disposal, j) products' bills of materials, k) turnover ratio of warehouses and retailers, l) minimum percentage of demand and returned products that has to be satisfied, m) upper and lower bounds for purchases and flow of materials, n) salvage value, tax and interest rates, o) time horizon and time slots' extent;

**subject to:** I) mass balance verification at each node of the network (plants, warehouses, retailers and markets) at each time slot, II) the capacity of each infrastructure cannot be exceeded, III) material flows between each pair of entities respect the imposed boundaries, IV) products' bill of materials must be complied.

It is desired to **determine** the following results that maximize the expected NPV: 1) design decisions, which include the number, location and capacity of the processes that have to be installed in each plant, warehouses and retailers, at each time period, and 2) planning decisions that are related to the production rates at plants, forward and reverse flows between the nodes of the network, inventory levels at warehouses and retailers, establishment of transportation links between all entities, also defined for each time period.

The problem was modeled through a Mixed Integer Linear Programming (MILP) that uses a time horizon discretization where all the design and planning constraints, defined above, were modeled. The model was built assuming that each decision will have effect during the complete time slot duration. Thus, design and planning decisions may only change at time slot boundaries.

Furthermore, the reverse flows consist of different types of products: (1) end of life products collected in time period  $t$ , that were sold to the market in a previous period  $T_n=t-n$ , where  $n$  accounts for the products' estimated lifetime; (2) non-conforming products, that are a fraction of the forward products supplied to markets; (3) returned products to be sent to disposal whose amount is calculated as a percentage of the total collection of returns at the end costumers.

### Model Formulation

Having in consideration the characteristics of the problem described, a MILP model was developed. However, due to the paper extent limitation, only the objective function is here presented. This is formulated by equation (1) that represents the maximization of the expected NPV that accounts for the expected net present value of each scenario  $s$  ( $NPV_s$ ) and the probability of the occurrence of each scenario ( $pb_s$ ). Equations (2) to (9) are support equations to calculate the  $NPV_s$ , as discussed below.

$$Max \sum_s pb_s \times NPV_s \quad (1)$$

Equation (2) calculates the  $NPV_s$  as the summation of the discounted cash flows ( $CF_{st}$ ) generated in each time period  $t$ , through the usage of the interest rate  $ir$ .

$$NPV_s = \sum_t \frac{CF_{st}}{(1 + ir)^{t-1}} \quad \forall s \quad (2)$$

Equation (3) is used to calculate the cash flow in each time period  $t$  obtained from the difference between the net earnings ( $NE_{st}$ ) and the fraction of the total depreciable

capital ( $FDC_t$ ) for all periods except for the last period. In Equation (4) it is calculated the cash flow for the last time period, where part of the total fixed capital investment (FCI) may be recovered, using a salvage value  $sv$ .

$$CF_{st} = NE_{st} - FDC_t \quad t = 1, \dots, NT - 1 \quad (3)$$

$$CF_{st} = NE_{st} - FDC_t + sv \times FCI \quad t = NT \quad (4)$$

The net earnings used in the previous equations are given by the difference between the incomes and the total cost, as it is shown in Equation (5). The incomes are determined from sales through the multiplication of the final products'  $p$  price ( $fp_{pvt}$ ) for the quantity that was delivered in each market  $v$  for each scenario node  $s$  ( $S_{vpst}$ ) in each period of time  $t$ . The total cost includes the purchases of raw materials that are calculated with the price of each raw material ( $rm_{pvt}$ ) and the quantity that was purchased by each entity  $v$  in each considered scenario tree node  $s$  and in each time period  $t$  ( $Pu_{pvst}$ ); the operational costs given by the manufacturing or disassembling cost of each process located at each entity ( $Oc_{ivt}$  or  $Ocd_{ivt}$ ) and the respective output flow in each scenario node ( $Out_{ivpst}$  and  $Outd_{ivpst}$ ). The inventory costs are obtained through the cost of having one product stored ( $Ci_{vt}$ ) multiplied by the average inventory level in each scenario node and time period ( $IL_{vst}$ ). Furthermore, the transportation costs between all the supply chain entities are the multiplication of the distance between each pair of entities ( $dt_{vw}$ ), the unitary transportation cost ( $dc_{vwt}$ ) and the respective flows in each scenario tree node  $s$ , which can be forward ( $Q_{vwpst}$ ) or reverse ( $QNC_{vwpst}$  and  $QEL_{vwpst}$ ).

$$NE_{st} = \left[ \sum_v \sum_p fp_{pvt} \times S_{vpst} - \sum_v \sum_p rm_{pvt} \times Pu_{pvst} - \sum_i \sum_v \sum_p Oc_{ivt} \times Out_{ivpst} - \sum_i \sum_v \sum_p Ocd_{ivt} \times Outd_{ivpst} \right]$$

$$- \sum_v \sum_w \sum_p dt_{vw} \times (Q_{vwpst} + QNC_{vwpst} + QEL_{vwpst}) \times dc_{vwt} - \sum_v Ci_{vt} \times IL_{vst} \Big] + (tr \times DP_t) \quad \forall s \quad (5)$$

The last term in Equation (5) describes the depreciation of the capital invested ( $DP_t$ ) multiplied by the tax rate ( $tr$ ), which is given by Equation (6) where the straight-line method is assumed. This technique assumes that during its useful life if the assets where the capital was invested, these assets had a cost that is divided equally over the considered period. The cost is equal to the original cost less the salvage value ( $sv$ ), which is an estimate of the value of the asset at the time it will be sold.

$$DP_t = \frac{(1 - sv)FCI}{NT} \quad \forall t \quad (6)$$

Equation (7) defines the total fixed cost investment (FCI), which is determined from the existing capacity of processes ( $im_{iv}$ ) and storage entities ( $is_v$ ) multiplied by the respective initial investment ( $in_{iv}^P$  or  $in_v^S$ ), variable ( $v_{ivt}^P$  or  $v_{vt}^S$ ) and fixed investments ( $f_{ivt}^P$  or  $f_{vt}^S$ ) associated to expansions of processes and storage capacities, respectively, as well as the establishment of transportation links during the entire planning horizon.

$$FCI = \sum_i \sum_v im_{iv} \times in_{iv}^P + \sum_v is_v \times in_v^S + \sum_i \sum_v \sum_t (v_{ivt}^P \times CE_{ivt}^P + f_{ivt}^P \times X_{ivt}^P) + \sum_v \sum_t (v_{vt}^S \times CE_{vt}^S + f_{vt}^S \times X_{vt}^S) + \sum_v \sum_w \sum_t lk_{vwt} \times (Y_{vwt} + YNC_{vwt} + YEL_{vwt}) \quad (7)$$

Finally, Eq. (8) assumes that the payment of the fixed capital investment is divided into equal sums for each time period.

$$FDC_t = \frac{FCI}{NT} \quad \forall t \quad (8)$$

## Model Implementation

The model was applied to an European case study where five distinct situations are explored prior to the analysis that considers uncertainty: (1) case A where the expansion of an existent supply chain is considered but where no reverse flows exist; (2) case B where the same supply chain is expanded but now reverse flows are considered with all the associated possible reverse logistics activities; (3) case C, an extension of case B where warehouses can directly deliver final products to the markets; (4) case D is also an expansion of the example used in case B where plants can directly send products to the markets; (5) case E is the most general example, where a supply chain with forward and reverse flows is considered and simultaneously plants, warehouses and retailers can send products to the markets.

The existent supply chain is characterized by one plant in Hamburg where twelve operational processes are installed, one warehouse in Munich, with six assembly lines for packing products and fifteen retailers that supply eighteen markets located in fifteen European countries. For all cases except A, where the reverse flow is not integrated, the existing plant has also six processes installed to disassemble and reprocess the returned products. It is also considered a demand increase of 3% in each time period.

Since this supply chain operates in a global scale it is extremely difficult to forecast the demand. So, for the most general supply chain structure (case E), it is analyzed the impact of uncertainty in the products' demand, which gives origin to case E\*. For this case there are used three different scenarios: 1) optimistic, which assumes that the demand is going to increase 5% in the following period, with a probability of 0.25; 2) realistic that considers an increase of 3% and has a probability of 0.5; 3) pessimistic, which assumes a decrease of 5%, with a probability of 0.25.

The possibility of installing new plants and new warehouses and choosing new retailers is thus studied under a time frame of three time periods, each one

representing 5 years. Figure 3 shows the location of all possible entities of the supply chain.



Figure 3. Location of supply chain entities

In order to compare all cases, the operational results obtained are presented in table 1.

Table 1. Operational results

(10 <sup>3</sup> €)	A	B	C	D	E	E*
Distribution	38126	34087	31902	41717	39416	38378
Production	1230	680	689	965	978	948
Purchasing	1049	529	533	880	884	855
Inventory	11.5	11.8	5.7	12	5.8	5.7
Sales	41687	44220	44602	60493	60784	5925
NPV	274.2	5229	6855	10153	11811	11549

When comparing case A with B, it is possible to conclude that with the integration of the reverse flows there is a decrease in the purchases of raw materials and in the manufacturing costs, since collected products are reutilized. There is also an increase in sales, fact that results in a significant augment in the net present value. Regarding cases B and C, it is important to notice that there is a decrease in transportation costs, because in the first case it is necessary to send the products from the plants to warehouses then to retailers and then to the markets and in case C markets are directly supplied from warehouses. Also a reduction in the inventory costs is observed since in case C the majority of the retailers act only as collecting centers. These receive the returned products, sort and send them to the respective infrastructure resulting in an increase of approximately 31% in the NPV. When comparing case C with case D, as it was expected there is an increase in all costs, since there is a new demand of intermediate products, which can be supplied directly from plants. However this is compensated

with an increase in sales, which results in an increment of, approximately, 41% in the NPV. Regarding cases D and E, there is a decrease in the inventory costs because the retailers do not keep inventory, resulting in an increase of 16.3% in the objective function.

Finally, we can see that the inclusion of uncertainty in the products' demand - case E\* - results in a decrease of 2.2% in the net present value essentially due to the reduction in the sales, which are, in this case, weighted in the expected NPV through the scenario probability.

The computational results of the model for all cases under study are presented in table 2. The model was implemented in GAMS 23.6, using CPLEX 12.0, in a Two Intel Xeon X5680, 3.33 GHz computer with 12 GB RAM.

Table 2. Computational results

	A	B	C	D	E	E*
# Total Variables	72816	449700	450672	453791	481938	4666247
# Binary Variables	1701	8181	9153	9201	9315	9315
#Restrictions	32116	63436	62680	62789	63004	270045
Gap (%)	0	0	0	0	0	0.03
CPU (s)	41.7	92.3	101.4	153.1	229.4	1848.6
Obj. Function (10 <sup>3</sup> €)	274.2	5229.5	6854.7	10153.5	11810.9	11549.1

As expected the increase in complexity by introducing the reverse flows and new possible links, results in an increase in the number of variables and constraints. However the gap for all is zero. The computational time suffered an increment with the problem complexity increased, but the model gives results in less than 4 minutes. When uncertainty is taken into account, in case E\*, there is an increment in the number of variables, constraints and computational time of one order of magnitude, since in this case they are considered 9 scenarios to capture demand uncertainty.

## Conclusions

In this work it is proposed an optimization model for the design and planning of a four echelon supply chain with forward and reverse flows. These allow for simultaneous incorporation of both production and delivery of products as well as reverse logistics activities

dealing with products non conformity as well as recovery of products in its end of life time. The model developed was applied to the design and planning of an European supply chain where different cases were studied. The model application shows that reverse logistics incorporation although costly may result in an increment of economical benefits associated with the recovering of products. Also greater flexibility in terms of flows tends to create a more profitable network. Finally, it was also shown that addressing uncertainty is essential so as to develop a robust network structure.

## Acknowledgments

The authors gratefully acknowledge financial support from Fundação para a Ciência e Tecnologia (FCT), grant SFRH/BD/64125/2009 and the project PTDC/SEN-ENR/102869/2008.

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