A PRIMAL DECOMPOSITION APPROACH FOR THE SUPPLY CHAIN INVESTMENT PLANNING PROBLEM UNDER DEMAND UNCERTAINTY

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Abstract

This paper presents the application of a primal decomposition algorithm for the problem of supply chain investment planning under uncertainty applied to the petroleum products supply chain. The uncertainty considered is related with the unknown demand levels for oil products. For this purpose, a model was developed based on two-stage stochastic programming. It is proposed two different solution methodologies, one based on the classical cutting plane approach presented by Van Slyke and Wets (1969), and other, based on a multi cut extension of it, firstly introduced by Birge and Louveaux (1988). The methods were evaluated on a real sized case study. Preliminary numerical results obtained from computational experiments are encouraging.

Keywords

Supply Chain Investment Planning, Stochastic Optimization, Primal Decomposition.

Introduction

Oil companies are global multinational organizations whose decisions involve a large number of factors related to the supply of raw materials, their processing and distribution. For companies with strongly diversified sources of petroleum supply, a long cast of products, and multiple markets, the advance planning of all activities along the supply chain is vital. Such planning includes the definition of production levels of oil (from oil fields and offshore platforms) and of petroleum products (from oil refineries), as well as the distribution among these refineries and to the final consumers of oil products. Major oil companies are characterized by integrated and verticalized activities, and the activities of refining and distributing oil products are characterized by low profit margins. Therefore, techniques for decision-making optimization are frequently used in the context of the oil supply chain.

The use of optimization techniques for supply chain design and planning has already been observed in the literature since the 1970's, especially the in seminal works of Geoffrion and Graves (1974). Vidal and Goetschalckx (1997) and Beamon (1998) present an extensive literature review on supply chain models. Although the research literature on the strategic modeling of supply chains is quite rich, few studies have included uncertainty mitigation in addition to other decisions of financial scope, such as commercialization income, market considerations and investment planning. According to Sahinidis (2004), the incorporation of uncertainty into planning models using stochastic optimization remains a challenge due to the large computational requirements involved.

For nearly 50 years, companies in the oil and chemical industries have led the development and use of mixed integer linear programming to support decision making at all levels of planning. An overriding feature in the oil industry is its wide range of uncertainties, typically related to the unpredictable levels of demand for refined products, fluctuations in prices in domestic and international markets and inaccuracies in the forecasted production of oil and gas. For this reason, many works have used techniques based on mathematical programming to support decision-making under uncertainty (Escudero, 1999; Dempster, 2000; Al-Othman, 2008; Khor, 2008)

Due to the great level of uncertainties taken into consideration, and the fact that the aforementioned problem is modeled as a mixed-integer linear program, it might become computationally infeasible to deal with a great number of scenarios by solving deterministic equivalents of the stochastic problems. Therefore, a decomposition approach might turn out to be a valid alternative as solution methodology.

The first approaches using decomposition schemes for stochastic programs were presented by Van Slyke and Wets (1969), a framework based on Benders decomposition (Benders, 1962) applied to two-stage stochastic problems, which became known as the L-Shaped method. Birge and Louveaux (1988) present an extension of the method presented by Van Slyke and Wets (1969), exploiting the structure of two-stage stochastic problems to place several cuts at once at each major iteration.

Cutting-plane schemes has been successfully used in solving large-scale problems since the pioneering paper of Geoffrion and Graves (1974), e.g., the uncapacitated network design problem with undirected. arcs (Magnanti, 1986), the stochastic transportation-location problems (Franca, 1982), the locomotive and car assignment problem (Cordeau, 2000; Cordeau, 2001), and the non-convex water resource management problem (Cai, 2001) to name a few.

The objective of this paper is present a mathematical model for the optimization of the supply chain investment planning problem applied to the petroleum products supply chain. Uncertainties related to product demand levels are considered, thus, the stochastic programming framework is adopted as modeling approach. Furthermore, it is shown an application of two primal decomposition techniques based on cutting plane approaches as solution technique. Experiments were performed in order to evaluate the efficiency of the proposed algorithms.

The paper is organized as follows: section 2 describes the proposed mathematical model; section 3 presents the traditional primal decomposition framework, while section 4 presents the multi cut framework; computational results are shown in section 5; Section 6 draws some conclusion.

Mathematical Model

Petroleum products supply chains are composed by several types of nodes and arcs. Nodes are different in a sense that they might represent refineries, international markets, distribution bases, and marine terminals. Arcs are the connections between the nodes, and might represent pipelines, roadways, waterways, and so forth.

The objective here is to choose, among some possible investments, which projects should be implemented in order to reach the best logistic efficiency. What we understand as the ideal logistic efficiency is the configuration that would provide the lowest combination of costs for the chain.

The system is subject to several costs. Costs are related with freight, product inventory, investments, and demand shortfall.

To address the problem in question, a two-stage stochastic model is proposed based on mathematical programming (Birge and Louveaux, 1997). The first-stage comprises the decisions of which projects to implement and when; the second-stage decisions are those relating to the flows of products, inventory levels, supply provided to each demand site, and supply levels at sources. The purpose of the model is to provide the optimal distribution of refined products to meet the demand of distribution bases, minimizing the logistics costs of this operation and maximizing revenue for retailing such products. Meeting the demand depends on the characteristics of the network operations, refinery availability and sources of production. The supply transportation is defined in conjunction with investment decisions, which are chosen from a predefined portfolio of possibilities and allocated over the planning horizon. The uncertainties in the model are related to the levels of demand for petroleum products in the distribution bases, which are modeled as random variables.

Notation

The notation to be used for the presentation of the mathematical model is presented below. For the sake of notational compactness, the domains of summations will be omitted, except when the summation is evaluated only on a subset of the natural domain. When there is no mention of this fact, its domain should be considered as the original set to which the index refers. In addition to that, we use bold caption to represent decision variable vectors.

Indexes

$i, j, l \in \mathcal{N}$	Locations
$p \in \mathcal{P}$	Products
$t \in \mathcal{T}$	Time period
$\xi \in \Omega$	Uncertainty realization

Sets

$\mathcal{B} \subseteq \mathcal{N}$	Subsets of distribution bases
$\mathcal{N}$	Locations
$\mathcal{P}$	Products
$\mathcal{S} \subseteq \mathcal{N}$	Subset of suppliers
$\mathcal{T}$	Time periods
Ω	Uncertainty possible realizations

Parameters

A_{ij}^0	Current arc capacity
A_{ij}	Additional arc capacity
C_{ij}^t	Transportation cost
$D_{jp}^t(\xi)$	Demand
H_{jp}	Inventory cost
K_{jp}	Max. number of tank rotations
M_{jp}^0	Current inventory capacity
M_{jp}	Additional inventory capacity
O_{ip}^t	Supply

S_{jp}^{t}	Shortfall cost
W_{jp}^{t}	Inventory investment cost
Y_{ij}^{t}	Arc investment cost

Variables

$x_{ijp}^{t}(\xi)$	Product flow
$v_{jp}^{t}(\xi)$	Inventory level
$u_{jp}^{t}(\xi)$	Unmet demand
y_{ij}^{t}	Arc investment decision
w_{j}^{t}	Location investment decision

Formulation

The mathematical model for the optimization of aforementioned problem can be stated as follows:

$$\min_{\mathbf{w},\mathbf{y}\in\{0,1\}} \quad \sum_{j,p,t} W_{jp}^{t} w_{jp}^{t} + \sum_{i,j,t} Y_{ij}^{t} y_{ij}^{t} + \mathcal{Q}(\mathbf{w},\mathbf{y}) \tag{1}$$

$$s.t. \quad \sum_{t} w_{jp}^{t} \leq 1 \qquad \forall j,p \tag{2}$$

$$\sum_{t} y_{ij}^{t} \leq 1 \qquad \forall i,j \tag{3}$$

where the term $\mathcal{Q}(\mathbf{w},\mathbf{y}) = \mathbb{E}[Q(\mathbf{w},\mathbf{y},\xi)]$ represents the expectation evaluated over all $\xi \in \Omega$ possible realizations for the uncertain parameters of the second-stage problem, given an investment decision $(\mathbf{w},\mathbf{y})$. Constraints 2 and 3 define that each investment can happens only once along the time horizon considered.

The second-stage problem $Q(\mathbf{w},\mathbf{y},\xi)$ can be stated as follows in Eqs. (4) to (9). The objective function (4) represents freight costs between the nodes, inventory costs, and the cost of shortfall. Equation (5) comprises the material balance in distribution bases. Constraint (6) limits the supply availability at sources. Constraint (7) defines the arc capacities and the possibility of its expansion through the investment decisions $\mathbf{y}$. In a similar way, constraint (8) defines the storage capacities together with its expansion possibility. Constraint (9) sets the throughput limit for bases, defined by the product of the available storage capacity with the maximum number of tank rotations.

$$\min_{x,u,v\in\mathbb{R}_+} \sum_{i,j,p,t} C_{ij}^{t} x_{ijp}^{t}(\xi) + \sum_{j,p,t} H_{jp} v_{jp}^{t}(\xi) + \sum_{j,p,t} S_{jp}^{t} u_{jp}^{t}(\xi) \tag{4}$$

$$s.t. \quad \sum_{i} x_{ijp}^{t}(\xi) + v_{jp}^{t-1}(\xi) + u_{jp}^{t}(\xi) = \sum_{l} x_{jlp}^{t}(\xi) + v_{jp}^{t}(\xi) + D_{jp}^{t}(\xi) \qquad \forall j \in \mathcal{B},p,t \tag{5}$$

$$\sum_{j} x_{ijp}^{t}(\xi) \leq O_{ip}^{t} \qquad \forall i \in \mathcal{S},p,t \tag{6}$$

$$\sum_{p} x_{ijp}^{t}(\xi) \leq A_{ij}^{0} + A_{ij} \sum_{t'\leq t} y_{ij}^{t'} \qquad \forall i,j,t \tag{7}$$

$$v_{jp}^{t}(\xi) \leq M_{jp}^{0} + M_{jp} \sum_{t'\leq t} w_{jp}^{t'} \qquad \forall j \in \mathcal{B},p,t \tag{8}$$

$$\sum_{i} x_{ijp}^{t}(\xi) \leq K_{jp}\left(M_{jp}^{0} + M_{jp} \sum_{t'\leq t} w_{jp}^{t'}\right) \qquad \forall j \in \mathcal{B},p,t \tag{9}$$

Primal Decomposition Method

The model proposed in the previous section can be defined as an optimization model with binary first-stage variables, continuous second-stage variables and discrete random parameters. Moreover, the model has relatively complete recourse (Birge and Louveaux, 1997) that is, for any feasible first stage solution, the second stage problem is feasible. Such characteristics allow us a primal decomposition framework based on Benders decomposition (Benders, 1962) applied to stochastic optimization.

We start by noting that the so-called *master problem* can be equivalently reformulated as follows:

$$\min_{\mathbf{w},\mathbf{y}\in\{0,1\}} \quad \sum_{j,p,t} W_{jp}^{t} w_{jp}^{t} + \sum_{i,j,t} Y_{ij}^{t} y_{ij}^{t} + M \tag{10}$$

$$s.t. \quad \sum_{t} w_{jp}^{t} \leq 1 \qquad \forall j,p \tag{11}$$

$$\sum_{t} y_{ij}^{t} \leq 1 \qquad \forall i,j \tag{12}$$

$$M \geq \mathcal{Q}(\mathbf{w},\mathbf{y}) \tag{13}$$

This formulation allows one to distinguish an important issue. Inequality (13) cannot be used computationally as a constraint, since it is not defined explicitly, but only implicitly, by a number of optimization problems. The main idea of the proposed primal decomposition method is to relax this constraint and replace it by a number of cuts, which may be gradually added following an iterative solving process. These cuts, defined as supporting hyperplanes of the second-stage objective function, might eventually provide a good estimation for the value of $\mathcal{Q}(\mathbf{w},\mathbf{y})$ in a finite number of iterations.

The primal decomposition method applied to the aforementioned problem can be stated as follows:

Initialization: Define LB and UB as lower and upper bounds. Set $LB = -\infty$ and $UB = \infty$. Define B as the iteration counter and set $B = 0$. Let $(\hat{\mathbf{w}},\hat{\mathbf{y}})$ denote the incumbent solution.

Step 1: Solve the *master problem* and let $(\mathbf{w}^{B},\mathbf{y}^{B})$ and LB be its optimal solution and optimal objective value respectively.

Step 2: For each realization $\xi \in \Omega$ solve the slave problem (4)-(9) stated before fixing $(\mathbf{w}^{B},\mathbf{y}^{B})$ and calculate the value for $\hat{\mathcal{Q}}(\mathbf{w}^{B},\mathbf{y}^{B})$ given by equation (14),

$$\hat{\mathcal{Q}}(\mathbf{w}^B,\mathbf{y}^B)=\sum_{\xi\in\Omega}P(\xi)Q(\mathbf{w}^B,\mathbf{y}^B,\xi) \quad (14)$$

where $P(\xi)$ is the probability of realization ξ occurs. Let $\mathcal{F}(\mathbf{w},\mathbf{y})$ represent the first-stage cost function and:

$$\mathcal{G}(\mathbf{w}^B,\mathbf{y}^B)=\mathcal{F}(\mathbf{w}^B,\mathbf{y}^B)+\hat{\mathcal{Q}}(\mathbf{w}^B,\mathbf{y}^B) \quad (15)$$

If $\mathcal{G}(\mathbf{w}^B,\mathbf{y}^B)<UB$ then update $UB=\mathcal{G}(\mathbf{w}^B,\mathbf{y}^B)$ and the incumbent solution $(\widehat{\mathbf{w}},\widehat{\mathbf{y}})=(\mathbf{w}^B,\mathbf{y}^B)$.

Step 3: If $UB-LB\le\epsilon$, where ϵ is a pre specified tolerance, then return the incumbent solution $(\widehat{\mathbf{w}},\widehat{\mathbf{y}})$ and UB as the objective function value. Otherwise, proceed to *Step 4*.

Step 4: Let $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\delta}$, and $\boldsymbol{\zeta}$ be the dual variables associated with constraints (5) to (9) respectively. Generate the cut (16):

$$M\ge\sum_{i,j,t}a_{ij}^t\left(\sum_{t'\le t}y_{ij}^{t'}\right)+\sum_{j,p,t}b_{jp}^t\left(\sum_{t'\le t}w_{jp}^{t'}\right)+K \quad (16)$$

Where:

$$a_{ij}^t=\sum_{\xi\in\Omega}P(\xi)A_{ij}\gamma_{ij}^t(\xi)$$

$$b_{jp}^t=\sum_{\xi\in\Omega}P(\xi)M_{jp}\left[\delta_{jp}^t+K_{jp}\zeta_{jp}^t(\xi)\right]$$

$$K=\sum_{\xi\in\Omega}P(\xi)\left[\sum_{j,p,t}\left(\alpha_{jp}^t(\xi)D_{jp}^t(\xi)+\beta_{jp}^t(\xi)O_{jp}^t+\delta_{jp}^t(\xi)M_{jp}^0+K_{jp}M_{jp}^0\zeta_{jp}^t(\xi)\right)+\sum_{i,j,t}\gamma_{ij}^t(\xi)A_{ij}^0\right]$$

Add the cut to the *master problem*. Update $B=B+1$ and go to *step 1*.

Multi Cut Primal Decomposition Method

The structure of stochastic programs allows one to add multiple cuts to the master problem instead of one in each major iteration. Birge and Louveaux (1988) show that the use of such a framework may greatly speed up convergence. The main idea behind this multi cut framework is to generate an outer linearization for all functions $Q(\mathbf{w},\mathbf{y},\xi)$, replacing the outer linearization of $\mathcal{Q}(\mathbf{w},\mathbf{y})$. The multi cut approach relies on the idea that using outer approximations of all $Q(\mathbf{w},\mathbf{y},\xi)$ send more information than the single cut on $\mathcal{Q}(\mathbf{w},\mathbf{y})$ and that, therefore, fewer iterations are needed. In fact, following Birge and Louveaux (1988), it is possible to show that the maximum number of iterations for the multi cut procedure is given by:

$$1+|\Omega|(q^m-1) \quad (17)$$

while the maximum number of iterations for the single cut procedure is given by:

$$[1+|\Omega|(q-1)]^m \quad (18)$$

where q represents the total of slopes for the second-stage problem and m the number of recourse constraints. Although q might turn out to be complicated to calculate for real world problems, bounds (17) and (18) show that the maximum number of iterations needed for reaching the optimum grows linearly with the number of realizations for the multi cut approach, while it grows exponentially for the traditional single cut approach.

Before stating the multi cut procedure, it is necessary to reformulate the original *master problem* to conveniently adequate it to the multi cut framework:

$$\min_{\mathbf{w},\mathbf{y}\in\{0,1\}}\sum_{j,p,t}W_{jp}^tw_{jp}^t+\sum_{i,j,t}Y_{ij}^ty_{ij}^t+\sum_{\xi\in\Omega}P(\xi)M(\xi) \quad (19)$$

$$s.t.\quad \sum_t w_{jp}^t\le 1 \qquad \forall j,p \quad (20)$$

$$\sum_t y_{ij}^t\le 1 \qquad \forall i,j \quad (21)$$

$$M(\xi)\ge \mathrm{Q}(\mathbf{w},\mathbf{y},\xi) \qquad \forall\xi \quad (22)$$

The main difference between the two approaches relies on the modification of *Step 4* from the single cut approach. The previous three steps should be considered as identical to those presented in the previous section. The modified *Step 4* is now stated as follows:

Step 4: Let $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, $\boldsymbol{\delta}$, and $\boldsymbol{\zeta}$ be the dual variables associated with constraints (5) to (9) respectively. Generate the group of cuts (23):

$$M(\xi)\ge\sum_{i,j,t}a_{ij}^t(\xi)\left(\sum_{t'\le t}y_{ij}^{t'}\right)+\sum_{j,p,t}b_{jp}^t(\xi)\left(\sum_{t'\le t}w_{jp}^{t'}\right)+K(\xi) \qquad \forall\xi\in\Omega \quad (23)$$

Where:

$$a_{ij}^t(\xi)=A_{ij}\gamma_{ij}^t(\xi)$$

$$b_{jp}^t(\xi)=M_{jp}\left[\delta_{jp}^t+K_{jp}\zeta_{jp}^t(\xi)\right]$$

$$K(\xi)=\sum_{j,p,t}\left(\alpha_{jp}^t(\xi)D_{jp}^t(\xi)+\beta_{jp}^t(\xi)O_{jp}^t+\delta_{jp}^t(\xi)M_{jp}^0+K_{jp}M_{jp}^0\zeta_{jp}^t(\xi)\right)+\sum_{i,j,t}\gamma_{ij}^t(\xi)A_{ij}^0$$

Add the cuts to the *master problem*. Update $B=B+1$ and go to *step 1*.

Numerical Experiments

In this section we describe numerical experiments using the proposed methodology for solving a realistic supply chain investment planning under demand uncertainty. The transport in the case study considered is primarily done using modal waterways, which are strongly affected by seasonality issues regarding the navigability of rivers. Four different products were considered - diesel, gasoline, aviation fuel and fuel oil - to be distributed over 19 locations (13 bases, 3 of which have sea terminals, one refinery and two external supply locations).

Waterway transportation is generally by large ferries, typically done during periods of river flooding and by smaller boats, which are able to navigate the sections during droughts, i.e., in periods of low water levels and higher transportation costs. The portfolio of projects considered for the study consists of 28 local projects and three arc project. Such projects are considered mutually independent and can therefore be combined as needed by the problem. The planning horizon considered was 8 years, divided into a total of 32 quarterly periods. Figure 2 illustrates the case study considered.

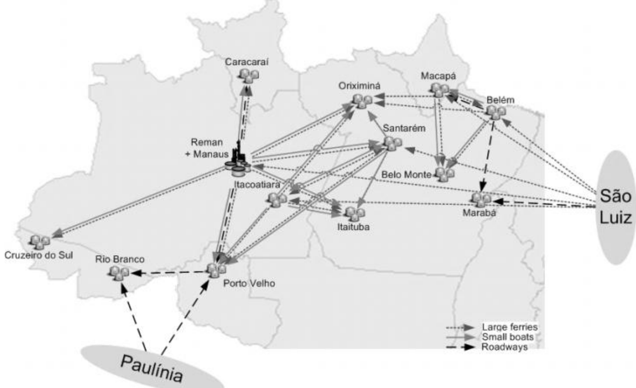

Figure 2 – Case study schematically represented

To take into account the uncertainty in demand levels for petroleum products, scenarios were generated by the following first order autoregressive model:

$$D_{jp}^{t} = D_{jp}^{t-1}\left[1 + \omega_p + \sigma\varepsilon\right] \qquad (24)$$

where ω_P represents the expected average growth rate for the consumption of product p over the planning horizon, σ represents the estimated maximum deviation for product consumption in the region and $\varepsilon \sim N(0,1)$. The maximum deviation was estimated based on the analysis of the annual consumption historical series over the last 40 years. Each scenario represents a possible product demand curve for the whole time horizon considered, for each product and place.

The mathematical model and the scenario generation routines were implemented using AIMMS 3.10. The mathematical model was solved using CPLEX 11.2. All experiments were performed on a Pentium Quad-Core 2.6 GHz with 8 Gb RAM. In AIMMS, an optimality parameter can be specified to decide whether to find the optimal solution or to quickly obtain a suboptimal solution, referred to as an ϵ-optimal solution. In these case studies, the execution of AIMMS was stopped when the value of the objective function was within 0.5% of the optimal solution, which is a reasonable choice in terms of solution accuracy. In addition, a time limit of 1 h (3600 s) was set. For the primal decomposition procedures, the tolerance ϵ was equivalently set as $\epsilon = 0.005(UB - LB)$, which is equivalent to define a 0.5% optimality tolerance. Table 1 summarizes the data of the experiments performed.

Table 1 – Experiment Summary

N	#Var	#Const.	DE(s)	SCut(s)	MCut(s)
20	194,443	204,024	18.20	56.08	12.25
30	291,243	306,024	29.81	41.14	28.52
40	388,043	408,024	40.92	45.70	24.98
50	484,843	510,024	48.34	84.42	45.53
60	581,643	612,024	86.31	113.92	51.17
70	678,443	714,024	160.84	101.30	70.75
80	755,243,	816,024	110.20	98.28	61.09
90	875,043	918,024	136.06	138.28	71.11
100	968,843	1.020,024	150.13	171.28	53.48

The first column of Table 1 represents the 9 different instances generated, with 20 up to 100 scenarios. The next two columns summarize the size of the complete model considering all scenarios at once, what is commonly known as the deterministic equivalent (Birge and Louveaux, 1997). It is worth to notice that all instances have the same number of integer variables, a total of 840 each.

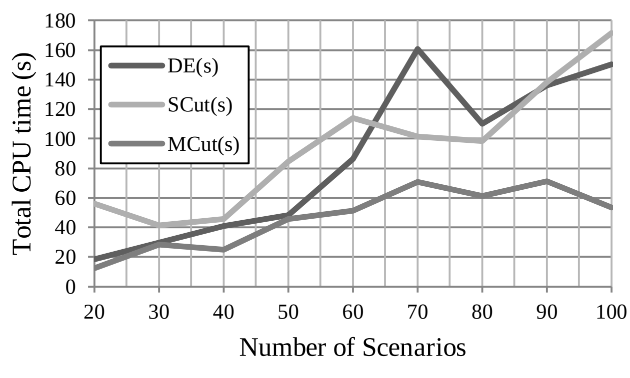

Figure 1 – Comparison of computation times

The last three columns from Table 1 show the solving time taken by each technique to reach the ϵ-optimal solution, namely solving the complete deterministic equivalent (DE), using the classical primal decomposition framework (SCut), and using the proposed multi cut approach (MCut). Figure 1 presents a graphical comparison among the three experiments regarding the CPU time required to reach the optimal solution.

As can be seen in Table 1, the multi cut approach has the smallest solution time for every instance, being up to 3 times faster than solving the deterministic equivalent and up to 5 times faster than using the single cut approach. Furthermore, it is worth to notice that the solution time for the single cut procedure is often higher than the solution of the deterministic equivalent among the experiments performed. This indicates that, for this particular case, it seems more efficient to simply solve the complete deterministic problem than use the traditional decomposition procedure.

Conclusions

This paper presents the application of a decomposition scheme for the problem of supply chain design applied to the petroleum byproducts supply chain. We propose a mathematical model that captures the impact of uncertainty on investment decisions, since the problem approached here is a mixture of logistic infrastructure investment planning problem and the stochastic transportation problem. With demand at each destination as a random variable, the objective is to minimize the sum of expected holding and shortage costs, transportation costs, fixed investment costs, and demand shortfall costs.

In order to solve the proposed model, we propose an application of a primal decomposition method to the problem at hand, together with the application of the multi cut extension of it, based on Birge and Louveaux (1988).

The results suggest that the first approach performs worse than the second in terms of computational time. It is an expected, yet important, result that corroborates the theoretical bounds for the total number of necessary iterations before complete convergence of the algorithms. In a general sense, the multi cut framework performs better than simply solving the deterministic equivalent - or even than directly applying the classic primal decomposition framework - allowing one to solve instances of greater size and, thus, with a more precise representation of the random variables.

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