MID-TERM PLANNING OPTIMIZATION MODEL WITH SALES CONTRACTS UNDER DEMAND UNCERTAINTY

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Abstract

Uncertainty modeling is a challenging topic in supply chain and operation management. When planning material purchase and stock levels, demand uncertainty could have an important impact on the plan results and its feasibility. Additionally, uncertainty could greatly affect customer satisfaction, inventory costs and company profits. From a modeling perspective, problems considering uncertainty are difficult to tackle and lead to complex optimization approaches. Due to those reasons, finding good models and methods to solve this type of problems has been a concern in this research field for the last decade. This work proposes a mid-term planning model dealing with sales contracts to diminish the effect of uncertainty. Another interesting feature considered in this work is given by the selection of different price levels. Price elasticity functions are introduced for each customer in order to jointly decide demand targets and prices. A linear generalized disjunctive programming model is developed. Short execution time shows that this model can be applied to analyze several real scenarios to decide material purchase plan, inventory levels, sales strategies, prices and demand levels in a medium term horizon planning.

Keywords

Demand Uncertainty, Planning, Contracts, Generalized Disjunctive Programming.

Introduction

Uncertainty modeling is a key issue in production and operation management for several reasons: first, uncertainty could have a great impact on customer satisfaction, inventory costs and company profits; second, it is difficult to tackle; and finally, optimization models considering uncertainty are complex, contain non-linearities in probabilistic approaches or have large size, especially in those cases dealing with multi-stage stochastic scenarios. Due to those reasons, finding good models and methods to solve this type of problems has been a concern in this research field for the last decade.

There are many parameters with a rather random behavior in real contexts, but demand is definitely one of the most challenging one (Rodriguez and Vecchietti, 2011). It is naturally exogenous and one of the main determinants of supply chain profits. One business practice in order to decrease demand uncertainty is to sign contracts with customers (Park, et al., 2006). In this way, customers beneficiate from better prices and financial conditions while companies assure certain demand level. In this work we analyze the effect of considering contracts with customer as a decision variable in a planning optimization model. In this approach, the model decides the set of customers to sign a selling contract according to their predicted behavior leaving the other clients with whole sales without contracts. Customers demand is modeled as continuous random parameter with normal distribution and known mean and standard deviation (Zipkin, 2000). In this problem, a target demand is considered which represents a level that the company wants to satisfy in the horizon terms. Then, if no contract is offered to customer, a minimum optimal safety stock level must be determined in order to accomplish that target.

Another interesting feature considered in this work is given by the selection of different price levels. Most planning optimization models from the literature consider that the product price is just one parameter that has previously been set by the company and a mean demand (and standard deviation) is known for that price. In this case, we introduce a demand elasticity function for each customer to model the relation between a set of possible prices and the corresponding mean demand levels. Price elasticity of demand, measures the responsiveness of the quantity demanded of a good or service when there is a change in its price (Browning and Zupan, 2011). Then, prices are also decisions in this approach which gives a more real insight of the offer-demand relationship in the decision process. This demand model is part of a wider planning model. We propose a generalized disjunctive program to select material and quantities to purchase in order to satisfy uncertain demand in a multi-period horizon planning.

Problem Formulation

The objective function is defined as the net present value during the planning horizon. Profits in each period are calculated according to the expected income, minus material purchases, lost sales costs and inventory holding costs. Materials *p* are grouped into families *f* in order to give flexibility to purchase decisions. In this way, different materials from a certain family can be used to satisfied product specifications .

$$NPV = \frac{\sum_k \sum_j Income_{jkt} - \sum_p m_{pt} - \sum_j \sum_k \sum_i Loss_i \cdot yz_{jkit} \cdot \sigma_{kj} \cdot lc_{kj} - \sum_f savg_{ft} \cdot ms \cdot cost\_avg_{ft}}{(1+i)^t} \quad (1)$$

Equation (2) shows that total initial stock of all families f in each time period t (s_{ft}) must be less than or equal to the material stock capacity CS. It is assumed that only raw materials are kept in stock, final products are manufactured according to customer orders.

$$\sum_f s_{ft} \leq CS \quad (2)$$

The stock at the beginning of each period is given by Eq. (3) as the initial stock in the previous period plus the materials bought from suppliers in that period $qf_{f(t-1)}$ minus the material consumption estimation $cons_{f(t-1)}$. This variable is determined by the maximum target demand the company aims to satisfy and the consumption of materials required to produce one unit of each product, as shown in Eq. (8).

$$s_{ft} = s_{f(t-1)} + qf_{f(t-1)} - cons_{f(t-1)} \quad (3)$$

Equation (4) establishes that in the first period, there is an initial stock given by Is_f.

$$s_{ft_1} = Is_f \quad (4)$$

Average stock level $savg_{ft}$ is estimated in Eq. (5) to determine the inventory costs in the objective function.

$$savg_{ft} = \frac{s_{ft} + qf_{ft} + s_{f(t+1)}}{2} \quad (5)$$

There is certain upper bound to each quantity q_{pt} that can be purchased of material *p* in period *t*. This limit is given by the suppliers' total capacity in Eq. (6).

$$q_{pt} \leq Qmax_p \quad (6)$$

Constraint of Eq. (7) determines the total amount bought of a material family qf_{ft} according to the materials that belong to that family.

$$\sum_{p \in FP_{fp}} q_{pt} = qf_{ft} \quad (7)$$

In order to calculate the consumption of material family in each period $cons_{ft}$ it must be taken into account a target demand that the company wants to satisfy. This corresponds to the objective sale OS_{jkt} for each product *k*, customer *j* in period *t*. The total raw material requirement for that OS_{jkt} is determined by Eq. (8) according to the unit material consumption α_{kf}.

$$\sum_j \sum_k OS_{jkt} \cdot \alpha_{kf} = cons_{ft} \quad (8)$$

Material purchase costs m_{pt} are calculated in the following Eq. (9) by multiplying the quantity ordered q_{pt} by the corresponding material cost $cost_{pt}$. Note that this parameter can vary during the horizon planning due to seasonal or inflation reasons.

$$m_{pt} \geq q_{pt} \cdot cost_{pt} \quad (9)$$

A price-demand discrete function is assumed for each product and customer. Figure 1 shows this situation for product p_1, which is a decreasing mathematical relationship between these two parameters. More elastic demand is shown when bigger changes in demand level are observed for each unit change in price. Once medium demand level μ^*_{jkt} is selected in Eq. (10), price is also determined. We have considered *h* possible demand and prices levels. This demand level is then used to calculate sales income in Eqs. (13) and (14).

$$\mu^*_{jkt} = \sum_h \mu_{jkh} \cdot yh_{jkht} \quad (10)$$

The following constraint in Eq. (11) determines that one demand level *h* has to be selected for each product and customer in each time period. Note that more than one product can be ordered by the same customer *j* and different price levels can be selected for each of them.

$$\sum_h yh_{jkht} = 1 \quad (11)$$

Disjunction (12) determines whether a contract will be offered to a customer for each product in each period time. If a contract is selected we assume demand uncertainty is negligible (i.e. σ_{jkt}~0) so objective sales must satisfy at least mean demand level μ^*_{jkt}. Taking into account that OS_{jkt} is applied to determine the amount of raw material required, no safety stock is required in this case. Otherwise, safety stock is required to address uncertain demand. This safety stock is determined indirectly by the additional demand ($z_{jkt}\ \sigma_{jkt}$) above the mean value selected in the second term of Eq. (12) where variable z_{jkt} represents the standard random variable with normal probability distribution. The amount of raw materials to satisfy this product demand is calculated in (8).

Expected income is determined in both cases. In the first one, we assume mean demand is served with the corresponding price and discount. In the second case, regular price is considered.

$$\begin{bmatrix} y_{jkt} \\ OS_{jkt} \geq \mu^*_{jkt} \\ Income_{jkt} \leq CI_{jkt} \end{bmatrix} \vee \begin{bmatrix} \neg y_{jkt} \\ OS_{jkt} \geq z_{jkt} \cdot \sigma_{jkt} + \mu^*_{jkt} \\ Income_{jkt} \leq NCI_{jkt} \end{bmatrix} \quad (12)$$

Contract income is determined in the constraint (13) considering the mean demand selected, the corresponding price and the discount offered to the customer. Note that since only one binary variable yh_{jkht} will be non-zero, then only one term in Eq. (13) will be positive.

$$CI_{jkt} = \sum_h \mu_{jkh} \cdot yh_{jkht} \cdot price_{hk} \cdot (1 - \delta_{hk}) \quad (13)$$

Similarly, Eq. (14) establishes the expected income when no contract is selected to a given customer and product. As mentioned, only one term will be non-zero on the right hand side of Eq. (14).

$$NCI_{jkt} = \sum_h \mu_{jkh} \cdot yh_{jkht} \cdot price_{hk} \quad (14)$$

Uncertain demand distribution is discretized using binary variable yz_{jkit}The normal standard variable z_{jkt} can assume *i* different values given by parameters ZP_i as shown in Eq. (15). This variable determines the additional demand for product *k*, customer *j* in period *t* that the company is willing to satisfy in case that no contract is signed.

$$z_{jkt} = \sum_i ZP_i \cdot yz_{jkit} \quad (15)$$

Equation (16) is a logical constraint establishing that if no contract is signed for a product *k* of customer *j* in period *t*, then one variable yz_{jkit} must be equal to 1 in order to determine variable z_{jkt} value. On the other hand, if a contract is selected (y_{jkt}=1) no yz_{jkit} is positive.

$$\sum_i yz_{jkit} = 1 - y_{jkt} \quad (16)$$

Equation (17) defines that certain service level sl_{jkt} must be satisfied if no contract is signed. This level is defined considering the cumulative probability ap_iassociated to the value selected of z_{jkt}which is given by yz_{jkit}.

$$\sum_i ap_i \cdot yz_{jkit} \geq sl_{jkt} \cdot (1 - y_{jkt}) \quad (17)$$

Results

The formulation was posed in GAMS system using LogMIP (Vecchietti and Grossmann, 1999) to model disjunctive terms. GDP problem is reformulated by LogMIP using convex hull relaxation which is solved as a MILP problem with CPLEX. The example considers 10 customers with 5 products to satisfy. For each product and customer, 5 price levels are evaluated with a Price-Demand discrete function as shown in Figure 1. Uncertain demand function is discretized using 17 points. The horizon planning is given by 4 months where 4 material families are handled to group 13 raw materials. The model size and performance are presented in Table 1.

Table 1. Model size and performance

OF: NPV	Equations	Positive variables	Binary variables	CPUs
376,170.54	2,377	1,373	4,600	17.51

Regarding model decisions, Figure 1 shows mean demand and prices selected with black circles, for product k_1 for each customer j in period t_2. This decision is solved for all products and periods but they are not shown due to space limitation. In most cases, lowest prices with highest demand levels are preferred. This is because demand functions present high elasticity values meaning that the percentage of increase in demand is greater than the percentage of decrease in price so incomes are maximized for highest demand levels. However, average stock cost and material purchase costs also increase when mean demand does. So under some circumstances, such as the cases of customers j_2, j_4, j_7 and j_8 of this example, a bit lower mean demand level is the best option.

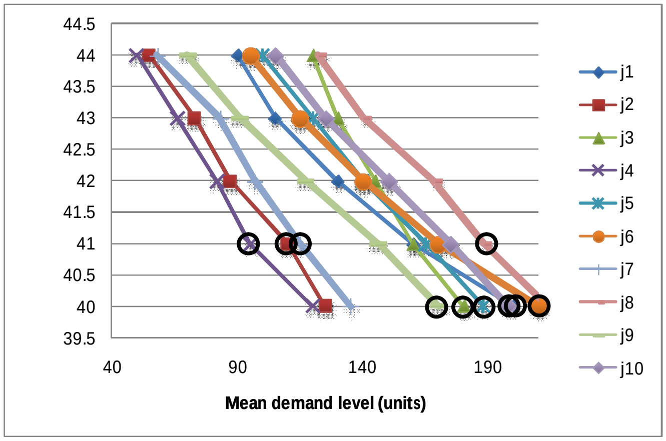


Figure 1. Mean demand and price selected for product k_1 in period t_2.

Another important result from the model is given by contract decisions. Table 2 shows to which customers j and for what products k the company should offer sales contract. Note that in most cases sale contract is selected. Even though they offer a discount to the customer, profits are increased because no safety stock or lost sales have to be faced when contract is signed. The elimination of demand uncertainty in these cases pays the decrease of income due to discount offered by the contract. On the contrary, in some cases like for product k_1, it is less profitable to offer a contract to the customers. The main reasons are that discounts offered are too high demand uncertainty is low (i.e. standard deviation is low) or safety factor required by the customer is also in a low level.

Table 2. Contract decision

		j_1				j_2				j_3				j_4				j_5			
	y_{jkt}	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
Product	k_1	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
	k_2	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes
	k_3	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_5	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	No	No	No	Yes	No	Yes	Yes	Yes	No	Yes	Yes	Yes

		j_6				j_7				j_8				j_9				j_{10}			
Product	k_1	No	No	No	No	No	No	No	No	No	No	No	No	No	Yes	No	No	No	No	No	No
	k_2	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_3	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_4	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	k_5	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes

For those cases where no contract is offered, safety stock must guarantee additional raw material availability in stock to cope with extra demand due to uncertain variations. These results are presented in Table 3. It is interesting to notice that safety stock level changes for each family and period to cope with demand variations when no contract is offered.

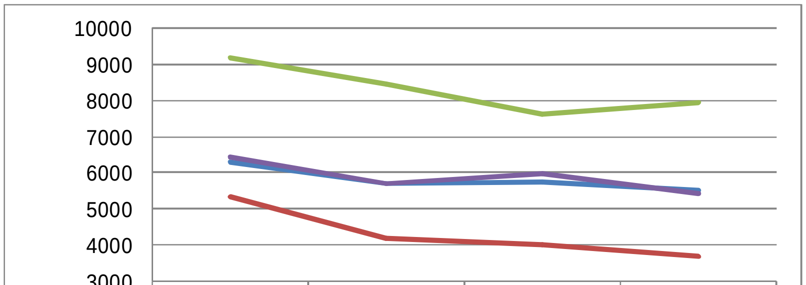
Table 3. Safety stock of material families

Family	Periods t_1	t_2	t_3	t_4
f_1	832.5	751.5	688	677
f_2	1158	580.5	402	262
f_3	2440	2082	1455	1738
f_4	481	317.5	417	193

Also lost sales costs are considered when no contract is offered to clients. This information is presented in Table 4.

Average stock levels for material families fluctuate in the planning horizon as shown in Figure 2. These value are calculated in the model according to material purchase, safety stock and consumption decisions. Note that if we compare safety stock from Table 3 with the average stock level form Figure 2 it can be clearly concluded that safety stock are actually a small proportion of this expected variable. This result shows that company decisions can definitely decrease demand uncertainty, which is one of the main positive effects of offering contracts to customers.

Regarding expected profits, its total value is $875,185 where $481,421 comes from contract sales while $393,764 comes from regular sales without contracts. Figure 3 shows that the proportion of contract income represents the 55% of total sales.



	t1	t2	t3	t4
f1	6287.75	5710.75	5764.75	5538
f2	5366.83	4186.83	4033.78	3723.4
f3	9177.2	8476.5	7625.55	7950.7
f4	6444.1	5720.75	6001.25	5419.5

Figure 2. Average material stock evolution

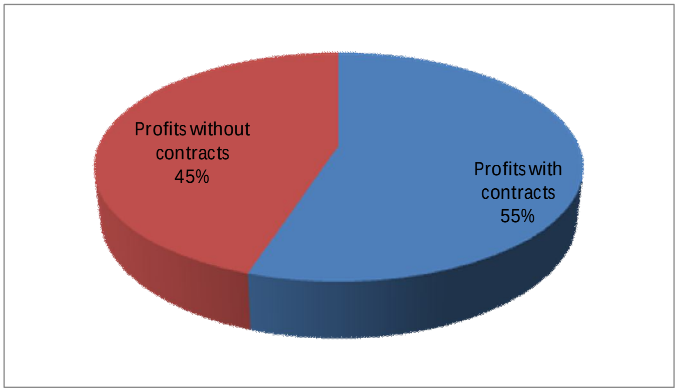


Figure 3. Profits obtained with regular and contract sales

Since no contract was chosen in the case of product k_1 bought by customer j_1 in the first period, mean demand was selected with 200 units but target demand is defined in 295 units. For this additional demand of 95 units, raw material must be stored as safety stock. If demand exceeds this target, lost sales will occur so expected lost sales are determined by the red area under the curve in Figure 4.

Table 4. Lost sales costs

		j_1				j_2				j_3				j_4				j_5			
		t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4	t_1	t_2	t_3	t_4
Product	k_1	158.3	158.3	375.8	158.3	118.7	17.58	118.7	49.98	66.64	66.64	66.64	66.64	138.5	20.51	138.5	58.31	49.98	49.98	49.98	49.98
	k_2					53.31	53.31	53.31	53.31					74.97	74.97	74.97	74.97				
	k_3					239.3		49.98						279.2							
	k_4																				
	k_5					83.3				83.3	83.3	83.3		446.8				99.96			
		j_6				j_7				j_8				j_9				j_{10}			
Product	k_1	149.9	149.9	356	149.9	134.5	19.92	134.5	56.64	73.3	73.3	73.3	73.3	296.7		296.7	125	83.3	83.3	83.3	83.3
	k_2	178				83.3	83.3	83.3													
	k_3					257.1															
	k_4																				
	k_5									94.96	94.96							99.96			

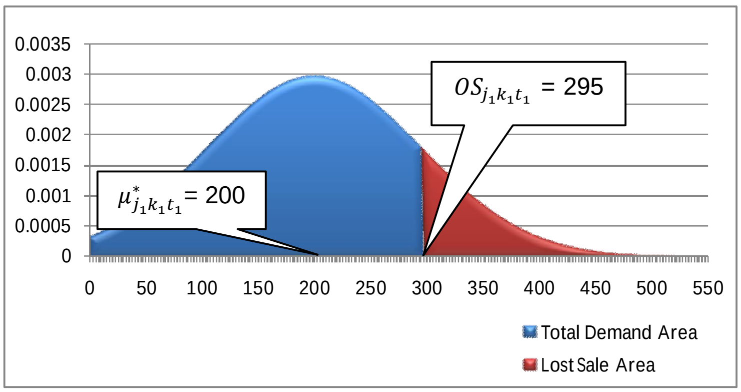


Figure 4. Demand decisions for customer j_1, product k_1 in period t_1.

Conclusions

The present work studies the effect of considering contracts with customers in order to address demand uncertainty. The model brings some interesting results that could help managers to make decisions regarding what level of demand is convenient according to price elasticity. It is also analyzed whether contract should be offered to customers in order to diminish the effect of uncertainty but offering the customer a discount over the regular price. When no contract is offered to a customer, it is considered that safety stock must be held to be able to satisfy the company demand target. Lost sales due to extra demand above that target level are also penalized in the objective function. Short execution time shows that this model can be applied to analyze several real scenarios to decide material purchase plan, inventory levels, prices and demand target in a medium term horizon planning.

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