INTEGRATION OF SCHEDULING AND CONTROLLER DESIGN FOR A MULTIPRODUCT CSTR

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Abstract

Though scheduling and control are traditionally considered separately due to the different operational levels and time scales, research on the integration of the two problems becomes attractive recently. It has been demonstrated that the integration can result in a collaborative operation with higher economic profits, which is crucial for the fierce competitions in current process industries. In this paper, we present a novel integration approach, which simultaneously determines the PI controller design and production scheduling decisions. The integrated problem is solved to update the controller parameters in each time slot and the control variable is then determined by the feedback of the process measurement according to the updated controller parameters. This feature is different from the previous integration strategies which determine the control variable directly from the solution of the integrated problem. The advantage of the proposed strategy is that the integrated problem can be solved in a large time scale while the controller works in real time. The two layer framework facilitates the online implementation of the integration of scheduling and control. The proposed approach is illustrated through an example of a multiproduct CSTR.

Keywords

Scheduling and control, PI controller, Decomposition algorithm, Cyclic production, Multiproduct CSTR

Introduction

Manufactures in the current process industry confront rapid market changes, stringent environmental policies, and aggressive rival competitions. In order to overcome the dwindling profit margins, it becomes increasingly important to optimize the global objectives by integrating the information and the decision-making among different operational levels (Grossmann 2004; 2005). Though scheduling and control are transitionally considered separately, a more economical operation can be achieved by taking them into account simultaneously (Flores-Tlacuahuac and Grossmann 2006; Harjunkoski et al. 2009; Munoz et al. 2011).

The process scheduling problem comprises the organization of human and technological resources, the determination of production sequence and production times. The process control problem aims to regulate the process variables and ensure the dynamic trajectories tracks the ones set to meet the operational criteria. In a multi-product manufacturing line, the transition period between two products is determined by the control system. Different designs of control systems will result in different values of process variables in the transition period, e.g. the transition material consumption and the transition time. These variables may significantly influence the scheduling decisions. On the other hand, to design a control system, the transition sequence and the set points are required to be determined from the scheduling problem. Besides, the economic objective function for used to design the control system design should be derived from the scheduling problem. The coupling of the scheduling problem and the control problem necessitates their integration.

The integrated problem of scheduling and control is frequently formulated as a mixed integer dynamic optimization (MIDO) (Barton et al. 1998; Allgor and Barton 1999). One solution approach is to transform the MIDO problem into a mixed integer nonlinear programming (MINLP) by using the collocation method (Cuthrell and Biegler 1987; Kameswaran and Biegler 2006). In this approach, the state trajectories and the input trajectories are discretized simultaneously and the differential equations describing the dynamic behavior of the process are transformed into a large set of nonlinear algebraic equations. This approach has been applied to the cyclic production in a single CSTR (Flores-Tlacuahuac and Grossmann 2006) and following extensions (Terrazas-Moreno et al. 2007; Flores-Tlacuahuac and Grossmann 2010).

An alternative to solving the MIDO problem is to use decomposition method (Nystrom et al. 2005; Prata et al. 2008). The scheduling problem is modeled as the master problem of MINLP while the control problem is formulated as the primal problem of dynamic optimization (DO). The MIDO problem is solved by iterations between the MINLP problem and the DO problem.

A challenge in most integration approaches is that the resulting control system from solving the MIDO problem is open loop, because the value of the control variable or the process input is determined by the optimization algorithm once and for all while the current measurement of the process variables are not taken into account. Due to the inevitable uncertainties and disturbances in the real process, an open loop control is scarcely applicable in practice. A new integration strategy which generates a closed-loop control system is proposed recently (Zhuge and Ierapetritou, 2011). The closed-loop control is realized by solving the integrated problem between the sampling duration of the measurement. This strategy is similar to the one adopted in model predictive control (MPC). However, solving the MIDO in real time is still formidable for most current solvers.

The challenge in the integration of scheduling and control arises from the different scales of the two problems in the time domain. The controller needs to work in real time to deal with the disturbance in the process immediately while the scheduling decisions have to be determined in a much larger time scale due to the complexity of the optimization problem involving discrete decision variables. Therefore, implementation of the integrated problem in the large time scale results in an open loop control problem while the implementation in real time makes the optimization problem formidable.

In this paper, we propose a novel integration approach to address these challenges. The main difference between the proposed strategy from the existing ones is that there is a closed loop controller for the process and the integrated problem is solved to determine the parameters of the controller instead of the direct value of the control variable. The value of the control variable is determined by the controller according to the measured process variables along with the controller parameters calculated from the integrated problem. This strategy employs the feature of different time scales for the scheduling and the control. The closed-loop controller works in real time while its parameters are updated by solving the integrated problem during the production period.

To solve the formulated MIDO, we propose a decomposition method. The MIDO is decomposed into a scheduling problem of MINLP and a series of control problems of DO in the transition periods. The proposed approach is illustrated through an example of a multiproduct CSTR.

Formulation of scheduling and control problems

This section presents the formulation of the scheduling problem and the control problem. The formulation varies according to different types of the production recipe and different dynamic behaviors of the process. A production model which is widely studied in the literature is the manufacture of multiple products in CSTRs (Nystrom et al. 2005; Flores-Tlacuahuac and Grossmann 2006; Mitra et al. 2011). Due to its popularity, the focus of this paper is placed on this type of model, which is, specifically, the cyclic production in a multiproduct CSTR (Pinto and Grossmann 1994; Flores-Tlacuahuac and Grossmann 2006).

In the production cycle displayed in Figure 1, a number of products are cyclically manufactured in a single CSTR. The cycle time is partitioned into slots and only one product is produced in one slot. Each slot is composed of two periods: the transition period and the production period. The product is produced only in the production period when the CSTR runs in the steady state. The transition period represents the changeover between different products in which the output of the CSTR varies with time.

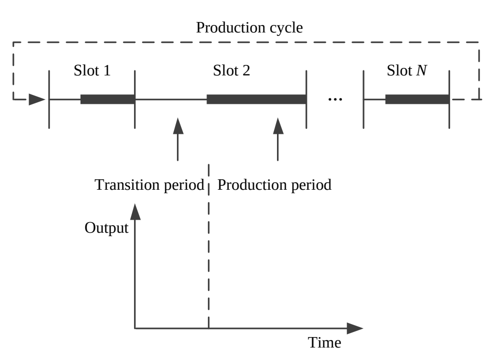

Figure 1: Cyclic production of a multiproduct CSTR

The scheduling problem aims to maximize the production profit by determining the sequence in which the products are manufactured as well as the production rate and the duration of each period in a time slot. Since the transition period is also dependent on the control system of the CSTR, it can achieve a better overall performance by integrating control problem into the scheduling problem than solving the two problems separately because tradeoffs can be made for conflicting objectives. The detailed formulations of the scheduling problem and the control problem are presented as follows.

Scheduling of cyclic production

Objective Function

$$\max\ \varphi = \varphi_1 - \varphi_2 - \varphi_3 \tag{1}$$

where φ_1 is the product revenue rate, φ_2 is the inventory cost rate, and φ_3 is the raw material cost rate. They are defined as

$$\varphi_1 = \frac{1}{T_c}\sum_{i=1}^{N} C_i^p W_i \tag{2}$$

$$\varphi_2 = \frac{1}{T_c}\sum_{i=1}^{N} \frac{1}{2} C_i^s \left(G_i T_c - W_i\right)\Theta_i \tag{3}$$

$$\varphi_3 = \frac{1}{T_c}\int_0^{T_c} C^r F\left(t\right) dt \tag{4}$$

Since the cyclic production is assumed to be carried out repeatedly, each term in the objective function is calculated as the averaged value in a production cycle. The cost of control in the transition periods is not formulated explicitly in the objective function, but considered implicitly in the cost of raw materials, which depends on the control system and tradeoffs other costs.

The objective function can include some penalty terms for the difference between the process variables and the set points (Flores-Tlacuahuac and Grossmann 2006). Though it is widely used in control system design, the objective function is difficult to be qualified economically and incorporated in the scheduling problem (Zhuge and Ierapetritou, 2011). Thus, we do not include it in the proposed model.

Product Assignment Constraints

$$\sum_{k=1}^{N} \xi_{ik} = 1,\ \forall i \tag{5}$$

$$\sum_{i=1}^{N} \xi_{ik} = 1,\ \forall i \tag{6}$$

$$\sum_{i=1}^{N} \beta_{ijk} = \begin{cases} \xi_{jN} & k=1 \\ \xi_{jk-1} & k\neq 1 \end{cases},\ \forall j,k \tag{7}$$

$$\sum_{j=1}^{N} \beta_{ijk} = \xi_{ik},\ \forall i,k \tag{8}$$

The binary variable ξ_{ik} denotes if the product i is assigned to the slot k. The number of slots is assumed to be equal to the number of products, denoted by N. Constraint (5) indicates that each product can be manufactured only once within a production cycle while constraint (6) implies that only one product is manufactured in each slot. The binary variable β_{ijk} denotes if product i is preceded by product j in slot k. The sequence variable β_{ijk} is related to the assignment variable ξ_{ik} according to constraints (7) and (8). Since the products are manufactured cyclically, the product manufactured in the first slot of a production cycle follows the one in the last slot, which is reflected by the equality of $\sum_{i=1}^{N}\beta_{ij1} = \xi_{jN}$ in constraint (7). As suggested in Wolsey (1997), the sequence variables can be replaced by continuous variables between 0 and 1, instead of binary variables. This significantly reduces the number of discrete variables and improves the computational efficiency.

Demand Constraints

$$W_i / T_c \geq D_i,\ \forall i \tag{9}$$

$$W_i = G_i \Theta_i,\ \forall i \tag{10}$$

The variable W_i denotes the total amount of the product i produced in each production cycle and its averaged value needs to meet the demand rate D_i, which is assumed to be constant. The production amount is determined by the multiplication of production rate G_i and production time Θ_i, according to the equation (10).

Timing Relation

$$t_k^e = t_k^s + \theta_k^t + \theta_k^p,\ \forall k \tag{11}$$

$$t_1^s = 0 \tag{12}$$

$$t_k^e = t_{k+1}^s,\ \forall k \neq N \tag{13}$$

$$t_N^e \leq T_c \tag{14}$$

$$\theta_k^p = \sum_{i=1}^{N} \theta_{ik},\ \forall k \tag{15}$$

$$\theta_{ik} \leq \theta^{\max} \xi_{ik},\ \forall i,k \tag{16}$$

$$\theta_k^t = \sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ijk}\tau_{ij},\ \forall k \tag{17}$$

$$\Theta_i = \sum_{k=1}^{N} \theta_{ik},\ \forall i \tag{18}$$

The slot k starts from time t_k^s to time t_k^e. The starting time of the first slot is set as zero while the end time of the last slot is bounded by the cyclic time T_c from above. Each slot consists of the transition period and the production period. The durations of the two periods are denoted by θ_k^t and θ_k^p respectively. The duration of the production period is related to the production time of the product i in the slot k in equation (15) and the equation (16) sets the upper bound for the production time. The duration of the transition period is dependent on the transition time from the product j to the product i, denoted by τ_{ij}, which is determined by the subsequent control problem. The equation (18) calculates the total production time of the product i, which is used to calculate the produced amount W_i in the equation (10) and the objective function φ_2 in the equation (3).

Control System in Transition Period

The product is manufactured only in the production period when the CSTR stays in the steady state. However, the transition between different products is dependent on the dynamic behavior of the CSTR. Information in the transition period, e.g. the transition time and the flow rate of the raw material, is required in the scheduling problem. To retrieve the information, the control system design for the CSTR is taken into account.

Suppose the dynamic behavior of the CSTR is described by a set of ordinary differential equations

$$\begin{cases} \dot{x}(t) = f\left(x(t), u(t)\right) \\ y(t) = g\left(x(t), u(t)\right) \end{cases} \tag{19}$$

where $x(t)$ contains the state variables, $y(t)$ denotes the process output, and $u(t)$ is the process input (the control variable).

The goal of control system design is to determine the value of $u(t)$ such that the output $y(t)$ has the desired properties. An objective function can be introduced to quantify the performance of an input profile and the optimal input can be calculated by minimizing this objective function

$$u(t)^* = \arg\max_{u(t)} \varphi_c\left(y_{sp} - y\left(t, u(t)\right), u(t)\right) \tag{20}$$

where φ_c is the objective function. In an output feedback control system, $u(t)$ is dependent on the control error which is the difference between the process output $y(t)$ and the set point y_{sp}. The dependence of the output $y(t)$ on the input $u(t)$ is determined by the system of equations (19).

It should be noted that the control system is designed on behalf of the scheduling problem in the upper level. So the objective function should be derived from the scheduling problem. Besides the objective function, the design procedure also requires other information from the scheduling problem, e.g. the production sequence and the set point of the output.

Since the control problem is coupled with the scheduling problem, the two problems can be solved simultaneously. Integration of scheduling and control results in an MIDO problem. A common solution is to discretize the differential equations by the method of simultaneous collocation (Flores-Tlacuahuac and Grossmann 2006; Nie and Biegler 2011).

The time domain is first discretized by a set of grid points $\{t_0, t_1, \cdots, t_{N_t}\}$. Then, in each interval of $[t_k, t_{k+1}]$, $k = 0, \cdots, N_t - 1$, the solution to the differential equation is approximated by a n-th order polynomial and the values of the state variables at the grid points are expressed by

$$x(t_{k+1}) = x(t_k) + h\sum_{i=1}^{n} b_i f_i \tag{21}$$

where $h = t_{k+1} - t_k$ is the interval length, b_i is the coefficient which is dependent on the type of polynomial used to approximate the solution, and f_i is the right hand side of the differential equation evaluated at the roots of the polynomial (collocation points).

Applying the collocation method, the differential equations (19) are transformed into a set of nonlinear algebraic equations (21) and the MIDO problem is reformulated as an MINLP problem.

Integration of PI Controller Design with Scheduling

The previous section presents the framework of formulating the scheduling and control problems. However, the strategy for integrating the two problems is not unique. This section starts from investigating some existing strategies presented in the literature and then proposes a new strategy which circumvents difficulties in the existing strategies. The resulted integration model is still an MIDO. Instead of the collocation method which solves the scheduling problem and the discretized control problem simultaneously, we propose a decomposition method, which separates the dynamic optimization in the control problem from the MINLP in the scheduling problem. The decomposition makes it possible to use different solvers for each problem and also provides an insight into the tradeoff between the two problems. The integrated problem is finally solved by iterations between the dynamic optimization and the MINLP.

Strategy for integrating scheduling and control

The first strategy for the integration is to simultaneously solve the two problems together (Flores-Tlacuahuac and Grossmann 2006) and then apply the calculated input to control the dynamic system in the transition period. The main deficiency of this integration strategy is that the control system is open-loop. Since the uncertainties and disturbances are inevitable in a control system, an open loop controller is scarcely applied in practice. Without the feedback, the control variable can only follow the specified value no matter what the output is.

To improve the deficiency, a closed-loop strategy is proposed (Zhuge and Ierapetritou, 2011). Instead of solving the integrated problem once, the problem is solved repeatedly between two sampling points of the process output. The real time information of the output can be taken into account to calculate the new input value. This strategy is similar to the one used in model predictive control (MPC) and it can significantly improve the performance of the control system since the uncertainties and disturbances are dealt with by the feedback. However, the main drawback of this strategy is the expensive computational efforts since the MIDO problem has to be solved on line.

To cope with the difficulty in the existing strategy, a new strategy is proposed in Figure 2.

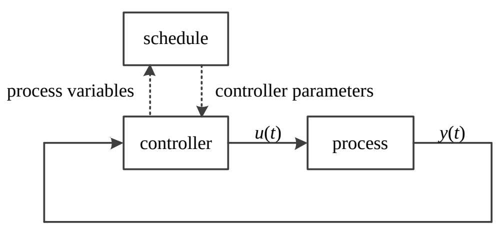

Figure 2: Strategy for integrating scheduling and control

The main difference between the proposed strategy from the existing ones is that there is a closed loop controller for the process and the integrated problem is solved to determine the parameters of the controller instead of the direct input value. The input value is determined by the controller according to the measured process variables along with the controller parameters determined in the integrated problem.

The advantage of this strategy includes:

(1) Fitting the practical implementation structure: Due to different time scales, scheduling and control are usually implemented at different levels. The real time controller is implemented at the lower level in a PLC while the scheduling algorithm is implemented at the higher level in a PC. The existing strategies for integrating scheduling and control break the common structure and a new system is required to build for the implementation. However, the proposed strategy can still work in the current structure. What needs to do is to transfer information about controller parameters and process variables between the two levels and this is easy to achieve by modern hierarchical control systems.

(2) Balancing the control system performance with the computational efforts: Since the integrated problem only determines the controller parameters, there is no need to solve the problem in real time. Communication between the scheduling algorithm and the controller can be done in a large time scale. For example, the control system can provide information of the current process variables at the end of the transition period and the integrated problem can be solved based on the current information during the production period. The new calculated controller parameters are then sent back to the controller at the beginning of the next transition period. Since the controller exists all the time, the control system is in the closed loop even if there is no communication between the upper level scheduling and the lower level controller.

The proposed integration strategy can be applied to any parameterized controller and the most widely used one is the traditional PID controller. The structure of the PID controller is shown in Figure 3. The controller is composed of three terms which are the proportional term, the integral term, and the derivative term of the control error. The input of the process system is the sum of the three terms. Since the derivative term will amplify the high frequency noise due to the differentiation operation, it is often omitted in practice and a PI controller is usually implemented.

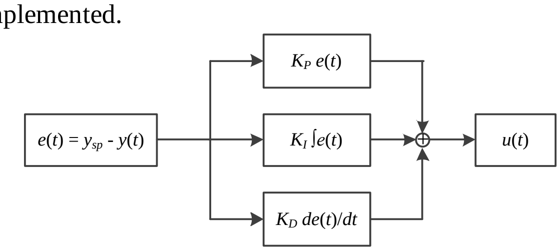

Figure 3: Structure of PID controller

The differential equation of a PI controller is

$$\begin{cases} \dot{x}_e(t) = e(t) \\ e(t) = y_{sp} - y(t) \\ u(t) = K_P e(t) + K_I x_e(t) \end{cases} \tag{22}$$

The controller has two parameters: the proportional gain K_P and the integral gain K_I. Though many methods exist to tune the two parameters, none of them take into account the objective function of the scheduling problem. Thus, we propose to determine these two parameters by solving the integrated problem of scheduling and control.

Algorithm for the integrated problem

Simultaneous discretization presented in the previous section is an alternative to solving the integration strategy proposed above. However, after discretization a large scale of nonlinear algebraic equations are derived. Suppose the number of grid points to discretize the time domain is N_t, and N_c collocation points are selected in each time segment between two adjunct grid points. There are totally $N_t N_c$ equations and variables derived by the collocation method. Moreover, this number is only for one state variable in one transition period. Considering the number of transition periods N and the number of states N_x, the total number of equations and variables generated by the discretization method is $N{\cdot}N_x{\cdot}N_t{\cdot}N_c$.

Furthermore, the collocation method is intrinsically an approximation method by polynomial and it is non-trivial to determine the grid points and the collocation. In the integrated scheduling and control problem, discretization points are probably required to be determined for each transition period. For a production cycle manufacturing N products, the number of possible transition periods is $N{\cdot}N{\cdot}N$ and the case-by-case investigation of each one is difficult.

Last, in the dynamic optimization problem for the proposed strategy, the controller parameters are calculated instead of the time function of the input. There is no need to discretize the decision variables.

Therefore, another alternative is adopted and a decomposition algorithm is proposed. To decompose the integrated problem, it is first required to analyze how the control problem entering the scheduling problem. The analysis can also provide an insight into the tradeoff between the two problems.

The control system only affects the variables in the transition periods, specifically the transition time θ_k^t and the transition flow rate of the material, defined as $F_k^t(t)$. The transition time is highly coupled with other variables in the scheduling problem while the transition flow rate only affects the objective function of the raw material cost. To separate the transition flow rate from other variables, the objective function (1) can be expressed as

$$\begin{aligned}&\varphi\left(z, F_1^t(t), \cdots, F_N^t(t)\right)\\ &=\varphi_1(z)-\varphi_2(z)-\varphi_3\left(z, F_1^t(t), \cdots, F_N^t(t)\right)\end{aligned} \tag{23}$$

where z is defined to include all decision variables except $F_1^t(t), \cdots, F_N^t(t)$. For the short notation, define

$$\left\{F_k^t(t)\right\} = F_1^t(t), \cdots, F_N^t(t) \tag{24}$$

By separating the transition flow rates, the objective function can be maximized by two steps, since

$$\max_{z,\left\{F_k^t(t)\right\}} \varphi \Leftrightarrow \max_z \max_{\left\{F_k^t(t)\right\}} \varphi \tag{25}$$

By expanding the expression of φ, the right side in the equivalence (25) turns out to be

$$\max_z\left(\varphi_1(z)-\varphi_2(z)-\min_{\left\{F_k^t(t)\right\}}\varphi_3\left(z,\left\{F_k^t(t)\right\}\right)\right) \tag{26}$$

or equivalently

$$\begin{aligned}&\max_z\left(\varphi_1(z)-\varphi_2(z)-\varphi_3\left(z,\left\{F_k^t(t)^*\right\}\right)\right)\\ &\text{s.t. } \left\{F_k^t(t)^*\right\} = \arg\min_{\left\{F_k^t(t)\right\}}\varphi_3\left(z,\left\{F_k^t(t)\right\}\right)\end{aligned} \tag{27}$$

By substituting the expression of φ_3, the inner minimization problem is

$$\begin{aligned}&\min_{\left\{F_k^t(t)\right\}}\frac{C^r}{T_c}\left(\sum_{k=1}^N\int_0^{\theta_k^t}F_k^t(t)\,dt+\sum_{k=1}^N\int_{\theta_k^t}^{\theta_k^t+\theta_k^p}F_k^p(t)\,dt\right)\\ &\Leftrightarrow \min_{F_k^t(t)}\int_0^{\theta_k^t}F_k^t(t)\,dt,\ \text{for each } 1\le k\le N\end{aligned} \tag{28}$$

where the transition flow rate and the production flow rate are defined as

$$F_k^t(t) = F\left(t+t_k^s\right),\ t\in\left[0,\theta_k^t\right] \tag{29}$$

$$F_k^p(t) = F\left(t+t_k^s\right),\ t\in\left[\theta_k^t,\theta_k^p\right] \tag{30}$$

The equivalence of the two problems in the expression (28) is due to the fact that in the inner minimization of φ_3 the variables except $\left\{F_k^t(t)^*\right\}$ are all regarded as parameters.

Therefore, the integrated problem can be solved by iterations between the scheduling problem and a series of control problems (Figure 4).

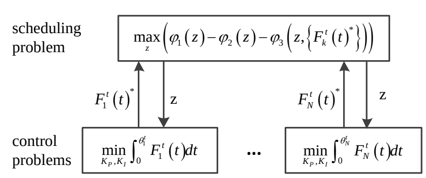

Figure 4: Decomposition of scheduling problem and control problem

The scheduling problem is formulated in the previous section, which can be summarized as

$$\begin{aligned}&\max\ \varphi && \text{Eq. }(1-4)\\ &\text{s.t.}\\ &\quad\text{Assignment constraints} && \text{Eq. }(5-8)\\ &\quad\text{Demand requirements} && \text{Eq. }(9-10)\\ &\quad\text{Timing relation} && \text{Eq. }(11-18)\end{aligned} \tag{31}$$

The closed-loop control problem including the process dynamic system and the controller is formulated as

$$I_F{}^* = \min_{K_P,K_I} x_F\left(\theta_k^t\right) \tag{32}$$

s.t.

$$\begin{cases} \dot{x}(t) = f\left(x(t), F_k^t(t)\right) \\ \dot{x}_e(t) = e(t) \\ \dot{x}_F(t) = F_k^t(t) \\ y(t) = g\left(x(t), F_k^t(t)\right) \\ e(t) = y_{sp} - y(t) \\ F_k^t(t) = K_P e(t) + K_I x_e(t) \end{cases} \quad (33)$$

$$e^{\min} \le e(t) \le e^{\max}, \quad t \ge \theta_k^t \quad (34)$$

$$F^{\min} \le F_k^t(t) \le F^{\max} \quad (35)$$

The information which the control problem requires is the value of the transition time θ_k^t and the transition sequence $j \to i$. Having the information, each control problem is a dynamic optimization and can be solved independently.

The dynamic optimization of the control system aims to minimize the integral transition flow rate. To evaluate the integral, a new variable of $x_F(t)$ is introduced, which denotes the integral of the transition flow rate from 0 to t. Consequently, a new differential equation of $x_F(t)$ is inserted. Besides this equation, the set of the differential equations (33) is derived by combining the system equations (19) and the equations describing the PI controller (22). The control variable $u(t)$ is substituted by the feed flow rate $F_k^t(t)$.

Under the PI controller, the process output will reach the set point after the transition period. The end of the transition period is indicated by the fact that the process output $y(t)$ constantly stays in a bound around the set point of y_{sp}, or equivalently the control error $e(t)$ falls into the interval bounded by $e^{\min}$ and $e^{\max}$ in the inequalities (34). The bounds of the control error are frequently set as ±2% of the set point. The dynamic model is simulated beyond the transition time θ_k^t while objective function of the control problem is only calculated up to the transition time. The last two inequalities (35) are set on the control variable, which is usually bounded due to the limited power of the practical instrument (saturation of the actuator).

Case Study

To illustrate the presented integration strategy of scheduling and control, the cyclic production in a multi-product CSTR is studied. The recipe and data of the model are given in Flores-Tlacuahuac and Grossmann (2006). The following reaction

$$3R \xrightarrow{k} P, \quad -r_R = kC_R^3 \quad (36)$$

takes place in an isothermal CSTR for manufacturing five products, A, B, C, D, and E. The dynamic model of the CSTR is described by

$$\frac{dC_R(t)}{dt} = \frac{F(t)}{V}\left(C_{in} - C_R(t)\right) + r_R \quad (37)$$

where C_{in} is the concentration in the feed flow and $C_R(t)$ is the concentration in the outflow. The flow rate $F(t)$ is the control variable. The design and kinetic parameters are C_{in} = 1 mol/L, V = 5000 L, k = 2 $L^2/(mol^2 h)$. Other data for the process are listed in Table 1.

Table 1. Process data for the case study

Type	F (L/hour)	C_R (mol/L)	G (kg/h)	Demand (kg/h)	Price ($/kg)	Inventory cost ($/kg)
A	10	0.097	9.03	3	200	1
B	100	0.200	80	8	150	1.5
C	400	0.303	278.7	10	130	1.8
D	1000	0.393	607	10	125	2
E	2500	0.500	1250	10	120	1.7

The objective is to maximize the product profit per hour. Decision variables include the production sequence, the production time and the transition time in each slot, and the controller parameters in each transition period. The MIDO of the integrated problem is formulated and solved by the decomposition method.

Table 2 lists the scheduling results obtained from the optimal solution to the integrated problem.

Table 2. Optimal scheduling result

Slot	Product	Production time (h)	Transition time (h)	Produced amount (kg/h)
1	A	31.27	1.00	3.00
2	E	22.54	0.80	300.27
3	D	1.55	1.08	10.00
4	C	3.36	3.32	10.00
5	B	9.38	19.52	8.00

The optimal production sequence is A→E→D→C→B. The produced amount of each product meets the demand. The cyclic time is 93.4 hour and the objective function is $\varphi = 40382.7 - 20531.7 - 6910.5 = 12940.5$ $/h.

Table 3. Controller parameters

Transition	K_P	K_I
A→E	5.0	14.9
E→D	5.4	19.9
D→C	4.1	19.0
C→B	4.5	18.0
B→A	2.5	1.7

The PI controller parameters in each transition period are determined from the integrated problem and listed in Table 3. It is seen that the controller parameters are set differently for different transition periods. This is an indicator that the control problem is dependent on the scheduling results. The dynamic profiles of the process input and the output for the closed-loop control system in each transition period are displayed in Figure 5.

The transition period starts at 0 and ends at the time marked by the vertical dash line. Due to the integral term in the PI controller, the control error will ultimately tend to zero and the process output can reach the set point in the steady state. To determine the transition time or the time the system enters into the steady state, two horizontal dash lines around the steady state are plotted. They are set as, respectively, ±2% of the steady state value. After the transition time, the process output stays between the two dash lines, indicating the end of the transition period.

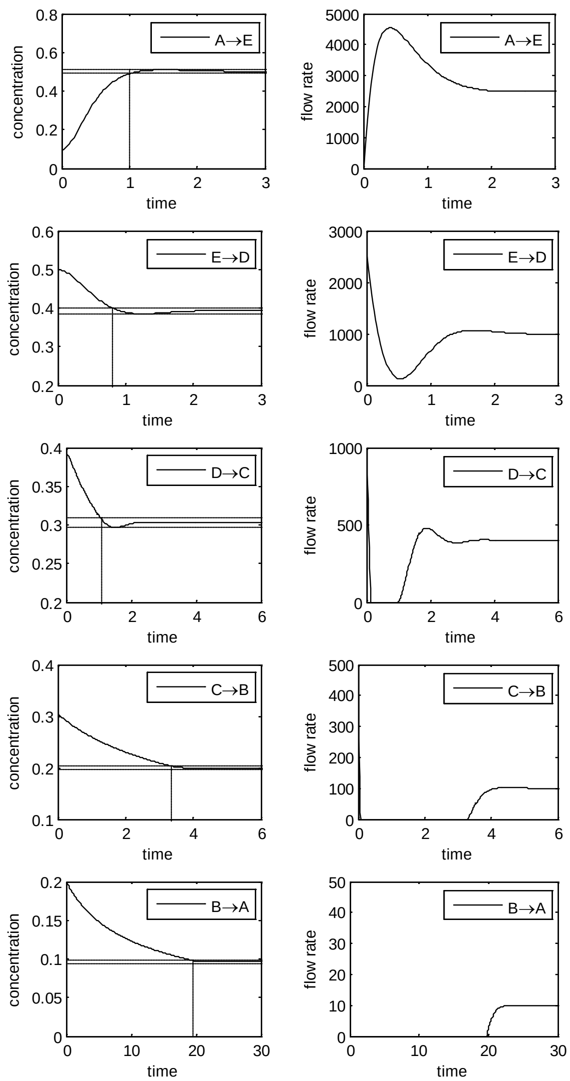

Figure 5. Optimal dynamic profiles of the concentration in the outflow (the process output in the left panel) and the feed flow rate (the control variable in the right panel) for each transition period

To compare the scheduling result, the production sequence is generated by a rule of thumb, i.e. A→B→C→D→E. The transition is made gradually from the product with the lowest concentration to the product with the highest concentration The scheduling results of the production sequence are calculated and listed in Table 4.

Table 4. Scheduling result of rule-of-thumb sequence

Slot	Product	Production time (h)	Transition time (h)	Produced amount (kg/h)
1	A	32.54	1.00	3.00
2	B	9.76	1.00	8.00
3	C	3.50	1.00	10.00
4	D	1.61	1.00	10.00
5	E	21.51	24.70	275.43

The cyclic time is 97.6 hour and the objective function is $\varphi = 37401.1 - 20249.5 - 6504.1 = 10647.5$ \$/h. The profit is much less than the one produced by the optimal solution.

Whenever a disturbance occurs in the process, the disturbance can be attenuated by the PI controller which works in real time. So there is no need to re-solve the whole integrated problem immediately. The policy adopted in the case study is that the integrated problem is re-solved at the beginning of each production period and the scheduling results as well as the controller parameters are returned to the controller before the start of the subsequent transition period. This means the integrated problem is only solved once in each production period and the requirement on the computational time is that the integrated problem can be solved within each production period, which is often large enough to do so.

To demonstrate this on-line rescheduling policy, a numerical experiment is conducted. The optimal solution to the integrated problem is applied and a disturbance is introduced in the transition period of A→E. The dynamic profiles of the process output and input are displayed in Figure 6.

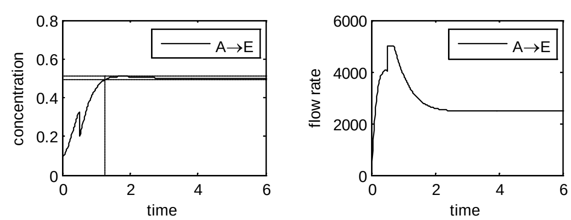

Figure 6. Dynamic profiles under the disturbance.

The disturbance occurs at the time of 0.5 hour and it is finally eliminated by the PI controller. The system enters into the steady state at the time of 1.26 hour which is larger than that without the disturbance. Though the transition time in this period is enlarged by the disturbance, the process can still successfully transfer to the production period. This is a significant difference from the open loop control where a slight disturbance can cause a failure in the production transition. This is also different from the real time rescheduling strategy since the controller parameters are still the ones calculated from the initial integrated problem and the integrated problem is not solved at this moment.

Table 5. Scheduling result under disturbances

Slot	Product	Production time (h)	Transition time (h)	Produced amount (kg/h)
1	A	31.44	1.26	3.00
2	E	22.54	0.80	298.68
3	D	1.55	1.08	10.00
4	C	3.38	3.32	10.00
5	B	9.43	19.52	8.00

The integrated problem is re-solved with the updated information from the controller from the beginning of the production period in which the product E is manufactured. The re-scheduling results are listed in Table 5.

It is seen that the production sequence is not affected by the disturbance. However the production time and the transition time are changed. The product profit is changed to $\varphi = 40192.1 - 20577.2 - 6979.1 = 12635.8$ \$/h, which is only a slightly lower than the optimal result without the disturbance.

Conclusion

A new strategy for integrating scheduling and control is presented for a multi-product CSTR. A PI controller is set to control the dynamic behavior of the process in the transition period between two products. The controller parameters are optimized simultaneously with the product scheduling problem. The controller works in real time while the integrated problem can be solved in a large time scale, e.g. within each production period. The presented integration strategy overcomes the difficulties in previous strategies in the literature. A tradeoff is made between the performance of the closed loop control system and the computational efforts for solving the integrated problem. The presented strategy is illustrated by a case study of a CSTR producing five products.

Nomenclature

Decision Variables

$e(t)$	control error
$e^{\max}$	upper bound on control error
$e^{\min}$	lower bound on control error
F	material flow rate (kg/h)
$F_k^p(t)$	flow rate in production period in slot k
$F_k^t(t)$	flow rate in transition period in slot k
$F^{\max}$	upper bound on transition flow rate
$F^{\min}$	lower bound on transition flow rate
G_i	production rate of production i (kg/h)
K_P	proportional gain of PID controller
K_I	integral gain of PID controller
K_D	derivative gain of PID controller
t_k^s	start time of slot k (h)
t_k^e	end time of slot k (h)
T_c	cycle time (h)
$u(t)$	control variable/process input
W_i	produced amount of product i (kg)
$x(t)$	state variable in process dynamic system
$x_e(t)$	integral of control error
$x_F(t)$	integral of transition flow rate
$y(t)$	process output
y_{sp}	set point of the output
z	all decision variables except the transition flow rate

Greek Letter

β_{ijk}	0-1 variable to denote if product i is preceded by product j in slot k
φ	objective function (\$/h)
τ_{ij}	transition time from product j to product i (h)
θ_{ik}	processing time of product i in slot k (h)
θ^{max}	upper bound on processing time (h)
θ_k^p	processing time in slot k (h)
θ_k^t	transition time in slot k (h)
Θ_i	total processing time of product i (h)
ξ_{ik}	0-1 variable to denote if product i is assigned to slot k

Parameters

C_i^p	price of product i (\$/kg)
C_i^s	inventory cost of product i [\$/(kg•h)]
C^r	raw material cost (\$/kg)
D_i	demand rate of product i (kg/h)
N	number of products/slots

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