Set-based adaptive estimation for a class of uncertain nonlinear systems: Application to parameter fault detection

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Abstract

In this paper, we consider the problem of parameter identification and state estimation of a continuous-time nonlinear system subject to exogenous disturbance. Using a set-based adaptive estimation, the parameters are updated only when an improvement in the precision of the parameter estimates can be guaranteed. The formulation provides robustness to parameter estimation error and bounded disturbance. The parameter uncertainty set and the uncertainty associated with an auxiliary variable is updated such that the set is guaranteed to contain the unknown true values. Two simulation examples are used to illustrate the effectiveness of the developed procedure and ascertain the theoretical results.

Keywords

Nonlinear observer, adaptive estimation, parameter estimation.

Introduction Parameter identification is an important problem in the theory of control systems. The exact of knowledge of process parameters can greatly influence the performance and robustness of control systems. In some case, the value of a process parameter can be associated with a particular mode of the process dynamics. In some cases, the value of the unknown parameters can be used to detect faulty operation in a process. The ability to estimate such parameters is therefore crucial for safe and productive operation of systems.

In almost all process applications, it is impossible to rely on a reliable measurement of all process variables. Some form of observer must be used to estimate the unmeasured variables. With respect to the parameter estimation problem, the knowledge of the unknown states may be necessary to provide precise estimates of the unknown parameters. In such cases, it is therefore necessary to consider the problem of simultaneous parameter identification and state estimation often referred to as adaptive observer design. Such observers are generally recognized to be useful in treating many practical problems such as fault detection, signal transmission or control, and, more recently, for synchronization of chaotic systems.

Several approaches have been proposed to simultaneously estimate the state and identify the parameters [Kreisselmeier, 1977] and [Zhu and Pagilla, 2003]. The basic idea in these approaches is to use a Luenberger observer [Luenberger, 1964] designed to operator in concert with a continuous parameter update law such that state observation error dynamics asymptotically approaches the origin. For both linear and nonlinear systems, one can show asymptotic and exponential convergence of the parameters to their true value subject to a persistency of excitation conditions. Some lower bounds of the rate of convergence, depending on the adaption gain and the level of excitation in the system, have been provided for specific control and estimation algorithms(e.g., [Kreisselmeier, 1977], [Sastry and Bodson, 1989] and [Marino and Tomei, 1995]). The derivation of parameter convergence rates remains an open area of investigation in adaptive systems design.

The present paper is inspired by the parameter identification scheme presented in [Adetola and Guay, 2010] and [Adetola, 2008]. The main contribution of this paper is to generalize this class of set-based parameter estimation scheme for the design of adaptive observers. The design technique consists of two parts. The first past is a set-based adaptive identifier for parameters (as proposed in [Adetola and Guay, 2009] and [Adetola and Guay, 2010]) that is suitable for estimation for a class of uncertain nonlinear systems. The method ensures convergence of the parameter to its true value provided the true parameters fall within an initial uncertainty set. In the second part, a Luenberger-like observer is chosen to ensure stability of the continuous-time error dynamics at the origin. Assuming that the initial conditions of state variables are contained in a known uncertainty set, a new set-based state estimation scheme is designed that ensures the non-exclusion of the true state from the uncertainty set. The

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proposed set-based estimation routine can be related to the class of interval observers proposed in [Gouzé et al., 2000] and more recently in [Mazenc and Bernard, 2011] and [Mazenc and Bernard, 2010]. The approach proposed in this paper leads to interval observers that take into account the impact of both disturbances and parameter estimation error. In this paper, we demonstrate how the set-based adaptive estimation technique leads to simple strategies for fault detection problems when faults can be associated with the value of the parameters.

This paper is organized as follows. The problem description is first given. State estimation and the corresponding uncertainty set adaptation are then described. The parameter estimation routine with uncertainty set update is presented. A brief description of the use the set-based technique to detect faults is discussed. This is followed by a simulation example and brief conclusions. The notation adopted in [Adetola, 2008] is used throughout this paper.

Problem Description

Consider a nonlinear system of the form

$$\begin{aligned} \dot{x} &= Ax + b(y)\theta + \omega(t) \\ y &= Hx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the vector state variables, $y \in \mathbb{R}^r$ is the vector output variables, $\theta \in \mathbb{R}^p$ is vector of unknown parameter.The vector-valued function $b(y)$ is assumed to be sufficiently smooth. The following assumptions are made about the nonlinear system (1):

Assumption 1 *The state variables $x(t) \in \mathbb{X}$ evolve on a compact subset of $\mathbb{R}^n$.*

Assumption 2 *The system is observable.*

Assumption 3 *It is assumed that θ to be uniquely identifiable lies within a known compact set $\Theta^0 = B(\theta_0, z_\theta)$, the ball centered at θ_0 is a nominal parameter value, with radius z_θ.*

Assumption 4 *The exogenous variable $\omega(t)$ is a bounded time-varying uncertainty disturbance that belongs to $\mathcal{L}_2$ (the space of square integrable functions). It is also assumed that $|\omega(t)| < \bar{\omega}$, a strictly positive constant.*

The objective of this work is to provide the true estimates of the plant parameters and estimate the state variables of the dynamical system in the presence of unknown bounded disturbances.

State and uncertainty set estimation

Let the estimator model for (1) be chosen as

$$\dot{\hat{x}} = A\hat{x} + b(y)\hat{\theta} + KHe + c^T\dot{\hat{\theta}}, \qquad K > 0, \tag{2}$$

$$\dot{c}^T = (A - KH)c^T + b(y), \qquad c(t_0) = 0. \tag{3}$$

Define the state prediction error as $e = x - \hat{x}$ and the auxiliary variable as $\eta = e - c^T\tilde{\theta}$. The error dynamics are given by:

$$\dot{e} = (A - KH)e + b(y)\tilde{\theta} - c^T\dot{\hat{\theta}} + \omega(t). \tag{4}$$

where $e(t_0) = x(t_0) - \hat{x}(t_0)$. The η dynamics are given by:

$$\dot{\eta} = (A - KH)\eta + \omega(t), \qquad \eta(t_0) = e(t_0) \tag{5}$$

As exogeneous disturbance $\omega(t)$ is not known, an estimate of η is generated by the system of differential equations

$$\dot{\hat{\eta}} = (A - KH)\hat{\eta}, \qquad \hat{\eta}(t_0) = \bar{x}(t_0) - \hat{x}(t_0) = \bar{e}(t_0). \tag{6}$$

where $\bar{x}(t_0)$ can be any initial state at a distance z_η (defined below) of the state estimates $\hat{x}(t_0)$. In general, $\bar{x}(t_0)$ can be taken as a worst case estimate of the initial conditions $x(t_0)$.

The resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics are given by

$$\dot{\tilde{\eta}} = (A - KH)\tilde{\eta} + \omega(t), \qquad \tilde{\eta}(t_0) = e(t_0) - \bar{e}(t_0). \tag{7}$$

We will need the following assumption concerning the initial error estimation error for η.

Assumption 5 *It is assumed that the initial estimation error $\tilde{\eta}$ is such that $\|\tilde{\eta}(t_0)\| \leq z_\eta$ where z_η is a known positive constant.*

Remark 1 *Since by construction $c(t_0) = 0$ then $\eta(_0) = e(t_0)$. Since $\hat{\eta}(t_0) = \bar{e}(t_0)$, one can interpret assumption 5 as a bound on the initial estimation error $\bar{x}(t_0) - x(t_0)$ considering the worst case estimate $\bar{x}(t_0)$. In general, one only needs to consider the case $\bar{x}(t_0) = 0$ and pick $\hat{x}(t_0)$ such that $\|\hat{x}(t_0)\| \geq \|x(t_0)\|$.*

As $w(t)$ is not known, an estimate of η is generated from (6) with resulting estimation error $\tilde{\eta} = \eta - \hat{\eta}$ dynamics given by (7), $\tilde{\eta}(t_0) = \tilde{\eta}^0 \in \chi^0$, where $\chi \triangleq B(0, z_\eta)$. In the following, the value of z_η is estimated using a set update algorithm. The next lemma is required to claim boundedness of the estimation error $\tilde{\eta}(t)$.

Lemma 1 *[Desoer and Vidyasagar, 1975] Consider the system*

$$\dot{x}(t) = Ax(t) + u(t)$$

Suppose the equilibrium state $x_e = 0$ of the homogeneous equation is exponentially stable. Then,

1. *if $u \in L_p$ for $1 < p < \infty$, then $x \in L_p$*
2. *if $u \in L_p$ for p= 1 or 2, then $x \to 0$ as $t \to \infty$.*

Consider Lyapunov function

$$V_\eta = \frac{1}{2}\tilde{\eta}^T P \tilde{\eta}. \tag{8}$$

It follows from (7) that

$$\dot{V}_\eta = \frac{1}{2}\tilde{\eta}^T P((A - KH)\tilde{\eta} + \omega(t)) + \frac{1}{2}((A - KH)\tilde{\eta} + \omega(t))^T P\tilde{\eta} \tag{9}$$

Using the following Ricatti equation

$$P(A - KH) + (A - KH)^T P = -Q, \tag{10}$$

one obtains,

$$\dot{V}_\eta = -\frac{1}{2}\tilde{\eta}^T Q\tilde{\eta} + \tilde{\eta}^T P\omega(t). \quad (11)$$

Note that one can always write:

$$\tilde{\eta}^T Q\tilde{\eta} \geq \lambda_{min}(Q)\tilde{\eta}^T\tilde{\eta} \geq 2\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}V_\eta. \quad (12)$$

By Young's Inequality

$$\tilde{\eta}^T P\omega(t) = \frac{k_1}{2}\tilde{\eta}^T P\tilde{\eta} + \frac{1}{2k_1}\omega(t)^T P\omega(t)$$
$$\leq k_1\lambda_{max}(P)V_\eta + \frac{\lambda_{max}(P)}{2k_1}\bar{\omega} \quad (13)$$

where $k_1 > 0$ is a positive constant to be assigned.

From (11), (12) and (13), the following inequality results:

$$\dot{V}_\eta \leq -\left(\frac{\lambda_{min}(Q)}{\lambda_{max}(P)} - k_1\lambda_{max}(P)\right)V_\eta + \frac{\lambda_{max}(P)}{2k_1}\bar{\omega} \quad (14)$$

Letting $k_1 = \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)^2}$ yields:

$$\dot{V}_\eta \leq -\frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}V_\eta + \frac{\lambda_{max}(P)^3}{\lambda_{min}(Q)}\bar{\omega} \quad (15)$$

This inequality will play an important role in the design of a set-based estimator for η. Considering (7), if $\omega(t) \in \mathcal{L}_2$, then $\tilde{\eta} \in \mathcal{L}_2$ (Lemma 1). Hence, the right hand side of (15) is finite.

Set adaptation for η An update law for the worst-case progress of the state in the presence of disturbance is given by

$$z_\eta = \sqrt{\frac{2V_{z\eta}}{\lambda_{min}(P)}} \quad (16)$$

$$V_{z\eta}(t_0) = \frac{1}{2}\lambda_{max}\left(P(t_0)\right)(z_\eta^0)^2 \quad (17)$$

$$\dot{V}_{z\eta} = -\frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}V_{z\eta} + \frac{\lambda_{max}(P)^3}{\lambda_{min}(Q)}\bar{\omega} \quad (18)$$

where $V_{z\eta}(t)$ represents the solution of the ordinary differential equation (18) with initial condition (17). The state uncertainty set, defined by the ball $\chi(0,z_\eta)$ is updated using (7) and the error bound (16) according to the following algorithm:

Algorithm 1 *Error bound z_η, the uncertain ball $\chi \triangleq B(0,z_\eta)$ is adapted on-line with algorithm:*

1. *Initialize $z_\eta(t_{i-1}) = z_\eta^0$,*
2. *At time t_i, update*

$$\chi = \begin{cases} \left(0,\chi(t_i)\right), \textit{ if } z_\eta(t_i) \leq z_\eta(t_{i-1}) \\ \left(0,\chi(t_{i-1})\right), \textit{otherwise} \end{cases}$$

3. *Iterate back to step 2, incrementing $i = i+1$.*

Algorithm 1 ensures that χ is only updated when z_η value has decreased by an amount which guarantees a contraction of the set. Moreover z_η evolution given as in (16) ensures non-exclusion of $\tilde{\eta}$ as given below.

Lemma 2 *The evolution of $\chi = B(0,z_\eta)$ under* (6),(16) *and Algorithm 1 is such that*

1. *$\tilde{\eta} \in \chi(t_0) \implies \tilde{\eta} \in \chi(t) \quad \forall t \geq t_0$*
2. *$\chi(t_2) \subseteq \chi(t_1), \qquad t_0 \leq t_1 \leq t_2$*

Proof:

1. We know $V_\eta(t_0) \leq V_{z\eta}(t_0)$ (by definition) and it follows from (15) and (18) that $\dot{V}_\eta(t) \leq \dot{V}_{z\eta}(t)$. Hence, we have

$$V_\eta(t) \leq V_{z\eta}(t) \qquad \forall t \geq t_0 \quad (19)$$

and since $V_\eta = \frac{1}{2}\tilde{\eta}^T P\tilde{\eta}$, it follows that

$$\tilde{\eta}(t)^T\tilde{\eta}(t) \leq \frac{2V_{z\eta}(t)}{\lambda_{min}(P)} = z_\eta^2(t) \quad \forall t \geq t_0. \quad (20)$$

Hence, if $\eta \in \chi(t_0)$, then $\eta \in B(\hat{\eta}(t), z_\eta(t)), \forall t \geq t_0$.

2. If $\chi(t_{i+1}) \not\subseteq \chi(t_i)$, then

$$\sup_{\tilde{\eta}\in\chi(t_{i+1})} \| \tilde{\eta}(t_i) \| \geq z_\eta(t_i) \quad (21)$$

Let $\lambda = \frac{\lambda_{min}(Q)}{2\lambda_{max}(P)}$. From the differential equation (18), it follows that at the time of the update one has:

$$V_{z\eta}(t_{i+1}) - V_{z\eta}(t_i) = \left(-1 + e^{-\lambda(t_{i+1}-t_i)}\right)V_{z\eta}(t_i) + \left(-1 + e^{-\lambda(t_{i+1}-t_i)}\right)\frac{\lambda_{max}(P)^3}{\lambda_{min}(Q)}\bar{\omega}$$

which indicates that

$$V_{z\eta}(t_{i+1}) - V_{z\eta}(t_i) \leq 0,$$

whenever

$$V_{z\eta}(t_i) \geq \frac{\lambda_{max}(P)^3}{\lambda_{min}(Q)}\bar{\omega}.$$

As a result, it follows that at the time of the update, it is guaranteed that:

$$V_{z\eta}(t_{i+1}) \leq \max\left[V_{z\eta}(t_i), \frac{\lambda_{max}(P)^3}{\lambda_{min}(Q)}\bar{\omega}\right]$$

or

$$z_\eta(t_{i+1}) \leq \max\left[z_\eta(t_i), \sqrt{\frac{2\lambda_{max}(P)^3}{\lambda_{min}(P)\lambda_{min}(Q)}\bar{\omega}}\right].$$

Since the algorithm 1 allows an update of the uncertainty set only when $z_\eta(t_{i+1}) \leq z_\eta(t_i)$, it follows that

$$\sup_{\tilde{\eta}\in\chi(t_{i+1})} \| \tilde{\eta}(t_i) \| \leq z_\eta(t_{i+1}) \leq z_\eta(t_i).$$

This proves that by construction one gets $\chi(t_{i+1}) \subseteq \chi(t_i)$. Hence, χ update guarantees $\chi(t_{i+1}) \subseteq \chi(t_i)$ and the strict contraction claim follows from the fact that χ is held constant over the update intervals $\tau \in (t_i, t_{i+1})$.

■

Parameter and uncertainty set estimation

Following [Adetola, 2008], the parameter estimation scheme has been generated for the above mentioned system.

Let $\Sigma \in \mathbb{R}^{n_\theta \times n_\theta}$ be generated from

$$\dot{\Sigma} = cH^T H c^T, \qquad \Sigma(t_0) = \alpha I \succ 0, \tag{22}$$

The preferred parameter update law, based on Equations (2),(3) and (6), as proposed in [Adetola and Guay, 2009] is given by

$$\dot{\Sigma}^{-1} = -\Sigma^{-1} cH^T H c^T \Sigma^{-1},\ \Sigma^{-1}(t_0) = \frac{1}{\alpha} I, \tag{23}$$

$$\dot{\hat{\theta}} = \text{proj}\left\{\Sigma^{-1} cH^T H(e-\hat{\eta}), \hat{\theta}\right\},\ \hat{\theta}(t_0) = \theta^0 \in \Theta^0, \tag{24}$$

where $\text{Proj}\{\phi,\hat{\theta}\}$ denotes a Lipschitz projection operator [M. Krstic and Kokotovic, 1995] such that

$$-\text{Proj}\{\phi,\hat{\theta}\}^T \tilde{\theta} \le -\phi^T \tilde{\theta}, \tag{25}$$

$$\hat{\theta}(t_0) \in \Theta^0 \implies \hat{\theta}(t) \in \Theta, \forall t \ge t_0 \tag{26}$$

where Θ^0 is initial uncertainty set. $\Theta \triangleq B(\hat{\theta}, z_\theta)$, where $\hat{\theta}$ and z_θ are the parameter estimate and set radius found at the latest set update respectively. The following Lemma will prove useful in the analysis of the estimation scheme proposed above. Note that the following notation is used:

$$\| \tilde{\eta} \|^2_{H^T H} = \tilde{\eta}^T H^T H \tilde{\eta},\ \| e-\hat{\eta} \|^2_{H^T H} = (e-\hat{\eta})^T H^T H(e-\hat{\eta}).$$

Lemma 3 *[Adetola and Guay, 2009] The identifier law* (23) *and parameter update law* (24) *is such that the estimation error* $\tilde{\theta} = \theta - \hat{\theta}$ *is bounded. Moreover, if*

$$\int_{t_0}^{\infty} \left[\| \tilde{\eta} \|^2_{H^T H} - \| e-\hat{\eta} \|^2_{H^T H}\right] d\tau < +\infty \tag{27}$$

and

$$\lim_{t\to\infty} \lambda_{min}(\Sigma) = \infty \tag{28}$$

are satisfied, then $\tilde{\theta}$ *converges to zero asymptotically.*

Proof: Let $V_{\tilde{\theta}} = \frac{1}{2}\tilde{\theta}^T \Sigma \tilde{\theta}$, it follows from (23), (24) that

$$\dot{V}_{\tilde{\theta}} = \tilde{\theta}^T cH^T H(e-\hat{\eta}) + \frac{1}{2}\tilde{\theta}^T cH^T H c^T \tilde{\theta} \tag{29}$$

Using the fact that $w^T \tilde{\theta} = e - \tilde{\eta} - \hat{\eta}$, one obtains:

$$\dot{V}_{\tilde{\theta}} \le -(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \tilde{\eta}^T H^T H(e-\hat{\eta}) + \frac{1}{2}(e-\hat{\eta}-\tilde{\eta})^T H^T H(e-\hat{\eta}-\tilde{\eta})$$

which is simply written as:

$$\dot{V}_{\tilde{\theta}} \le -\frac{1}{2}(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \frac{1}{2}\tilde{\eta}^T H^T H \tilde{\eta} \tag{30}$$

Since the estimation error $\tilde{\eta}$ is bounded, the last inequality implies that $\tilde{\theta}$ is bounded. Moreover, it follows from (30) that

$$V_{\theta}(t) = V_{\tilde{\theta}}(t_0) + \int_{t_0}^{t} \dot{V}_{\tilde{\theta}}(\tau) d\tau \tag{31}$$

$$\le V_{\tilde{\theta}}(t_0) - \frac{1}{2}\int_{t_0}^{t} \| e-\hat{\eta} \|^2_{H^T H} d\tau + \frac{1}{2}\int_{t_0}^{t} \| \tilde{\eta} \|^2_{H^T H} d\tau \tag{32}$$

Considering the dynamics of (7), if $\omega(t) \in \mathcal{L}_2$, then $\tilde{\eta} \in \mathcal{L}_2$ (Lemma 1). Hence, the right hand side of (32) is finite in view of (27), and by (28) we have $\lim_{t\to\infty} \tilde{\theta}(t) = 0$. ■

An update law that measures the worst-case progress of the parameter identifier in the presence of a disturbance is given by

$$z_\theta = \sqrt{\frac{V_{z\theta}}{2\lambda_{min}(\Sigma)}} \tag{33}$$

$$V_{z\theta}(t_0) = 2\lambda_{max}\Big(\Sigma(t_0)\Big)(z^0_\theta)^2 \tag{34}$$

$$\dot{V}_{z\theta} = -\frac{1}{2}(e-\hat{\eta})^T H^T H(e-\hat{\eta}) + \frac{1}{2}\lambda_{\max}(H^T H) z^2_\eta \tag{35}$$

where $V_{z\theta}(t)$ represents the solution of the ordinary differential equation (35) with the initial condition (34). The parameter uncertainty set, defined by the ball $B(\hat{\theta}_c, z_c)$ is updated using the parameter update law (24) and the error bound (33) according to the following algorithm:

Algorithm 2 *1. Initialize* $z_\theta(t_{i-1}) = z^0_\theta, \hat{\theta}(t_{i-1}) = \hat{\theta}^0$

2. At time t_i*, update*

$$\Big(\hat{\theta}, \Theta\Big) = \begin{cases} \Big(\hat{\theta}(t_i), \Theta(t_i)\Big), & \text{if } z_{\theta(t_i)} \le z_\theta(t_{i-1}) - \| \hat{\theta}_i - \hat{\theta}(t_{i-1}) \| \\ \Big(\hat{\theta}(t_{i-1}), \Theta(t_{i-1})\Big), & \text{otherwise} \end{cases}$$

3. Iterate back to step 2, incrementing $i = i+1$.

Algorithm 2 ensures that Θ is only updated when the value of z_θ has decreased by an amount which guarantees a contraction of the set. Moreover z_θ evolution as given in (33) ensures non-exclusion of θ as given below.

Lemma 4 *The evolution of* $\Theta = B(\hat{\theta}, z_\theta)$ *under* (23)*,* (33) *and Algorithm 2 is such that*

1. $\Theta(t_2) \subseteq \Theta(t_1), \qquad t_0 \le t_1 \le t_2$
2. $\theta \in \Theta(t_0) \implies \theta \in \Theta(t) \quad \forall t \ge t_0$

Proof:

1. If $\Theta(t_{i+1}) \not\subseteq \Theta(t_i)$, then

$$\sup_{s\in\Theta(t_{i+1})} \| s - \theta(t_i) \| \ge z_\theta(t_i) \tag{36}$$

However, it follows from triangle inequality and Algorithm 3.1 that Θ, at the time of update, obeys

$$\begin{aligned} \sup_{s\in\Theta(t_{i+1})} \| s - \hat{\theta}(t_i) \| &\le \sup_{s\in\Theta(t_{i+1})} \| s - \hat{\theta}(t_{i+1}) \| \\ &\quad + \| \hat{\theta}(t_{i+1}) - \hat{\theta}(t_i) \| \\ &\le z_\theta(t_{i+1}) + \| \hat{\theta}(t_{i+1}) - \hat{\theta}(t_i) \| \\ &\le z_\theta(t_i), \end{aligned}$$

which contradicts (36). Hence, Θ update guarantees $\Theta(t_{i+1}) \subseteq \Theta(t_i)$. And Θ is held constant over update intervals $\tau \in (t_i, t_{i+1})$.

2. We know that $V_{\tilde{\theta}}(t_0) \leq V_{z\theta}(t_0)$ (by definition) and it follows from (30) and (35) that $\dot{V}_{\tilde{\theta}}(t) \leq \dot{V}_{z\theta}(t)$. Hence, by the comparison lemma, we have

$$V_{\tilde{\theta}}(t) \leq V_{z\theta}(t) \qquad \forall t \geq t_0 \tag{37}$$

and since $V_{\tilde{\theta}} = \frac{1}{2}\tilde{\theta}^T \Sigma \tilde{\theta}$, it follows that

$$\tilde{\theta}(t)^T \tilde{\theta}(t) \leq \frac{2V_{z\theta}(t)}{\lambda_{min}(\Sigma(t))} = 4z_{\theta}^2(t) \quad \forall t \geq t_0. \tag{38}$$

Hence, if $\theta \in \Theta(t_0)$, then $\theta \in B(\hat{\theta}(t), z_{\theta}(t)), \forall t \geq t_0$.

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Application for fault detection

One of the properties of parameter update laws of the form proposed in this work is that one can extract from the bounds z_η and z_θ a bound on the estimation error $\|e\|$. By definition, one can write:

$$(y - \hat{y}) = He = H\eta + Hc^T\tilde{\delta}.$$

Thus, the error e is bounded as follows:

$$\begin{aligned} \|He\| &\leq \|H\eta\| + \|Hc\|\tilde{\delta} \leq \|H\tilde{\eta}\| + \|H\hat{\eta}\| + 2\|Hc\|z_{\theta 0} \\ &\leq \|H\|z_\eta + \|H\hat{\eta}\| + 2\|Hc\|z_{\theta 0}. \end{aligned} \tag{39}$$

The bound (39) is computable. It can be used to detect abnormal conditions. Since it is guaranteed by (39) that the true state value must within a ball of radius $z_e = \|H\|z_\eta + \|H\hat{\eta}\| + 2\|Hc\|z_{\theta 0}$ centered at the origin, any change in the process conditions such as sudden changes in the parameter values will cause a violation the inequality (39).

The strategy used for the detection and isolation is primarily on a generalization of algorithm 2. The main difference is that one checks if the inequality (39) is fulfilled. If it is not then one resets the algorithm to the initial conditions $(\theta_0, z_{\theta 0}, z_{\eta 0})$. Re-initiation of the algorithm allows one to re-estimate the states and the parameters corresponding to the new conditions. The value of $z_{\theta 0}$ must be chosen large enough to ensure that the new value of the parameter is contained inside the new uncertainty set. The value of $z_{\eta 0}$ must also be reset to a large value. In the present work, we simply set the value of z_η to the original value.

In summary, the set-based update algorithm with fault detection and estimation can be stated as follows.

Algorithm 3 *1. Intialize* $z_\theta(t_{i-1}) = z_\theta^0, \hat{\theta}(t_{i-1}) = \hat{\theta}^0$

2. *If* $\|e(t_i)\| > z_\eta(t_i) + \|\hat{\eta}(t_i)\| + 2\|c(t_i)\|z_\theta(t_{i-1})$, *increase* z_θ *to arbitrarily large value to keep the true parameter inside the uncertainty set.*

3. *At time* t_i, *update*

$$\left(\hat{\theta}, \Theta\right) = \begin{cases} \left(\hat{\theta}(t_i), \Theta(t_i)\right), & \text{if } z_\theta(t_i) \leq z_\theta(t_{i-1}) \\ & \quad - \| \hat{\theta}_i - \hat{\theta}(t_{i-1}) \| \\ \left(\hat{\theta}(t_{i-1}), \Theta(t_{i-1})\right), & \text{otherwise} \end{cases}$$

4. *Iterate back to step 2, incrementing* $i = i + 1$.

Simulation Example

We consider the population model presented in [Gouzé et al., 2000]. The dynamics are given by:

$$\begin{aligned} \dot{x}_1 &= -\beta_1 x_1 + \frac{\theta_1 x_3}{b + x_3} + \nu_1 \\ \dot{x}_2 &= \alpha_1 x_1 - \beta_2 x_2 + \nu_2 \\ \dot{x}_3 &= \alpha_2 x_2 - \beta_3 x_3 + \nu_3 \\ y &= x_3 \end{aligned}$$

where $\nu = [\nu_1, \nu_2, \nu_3]^T$ with $\nu_1 = 0.001\sin(0.01t)$, $\nu_2 = 0.001\sin(0.005t)$ and $\nu_3 = 0.001\sin(0.1t)]^T$.

It is assumed that $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$ and b are known constants where $\alpha_1 = \alpha_2 = 0.8$, $\beta_1 = \beta_2 = 1$, $\beta_3 = 0.5$ and $b = 1$. The nominal value of $\theta_1 = 1$.

Remark 2 *Note that the estimation of the constant b appearing in the nonlinear term can also be estimated using similar techniques. However the treatment of nonlinear parameterization is outside the scope of this manuscript.*

Following the theory, we identify the matrix

$$A = \begin{bmatrix} -\beta_1 & 0 & 0 \\ \alpha_1 & -\beta_2 & 0 \\ 0 & \alpha_2 & -\beta_3 \end{bmatrix}$$

and the output injection nonlinearity as $b(y) = \left[\frac{x_3}{b+x_3}, 0, 0\right]^T$.

We consider the following observer gain $K = [5, 5, 5]$. We consider a matrix $Q = 1I_{3\times 3}$ where $I_{3\times 3}$ represents the 3×3 identity matrix. These design choices yield the matrix

$$P = \begin{bmatrix} 0.625 & 0.3125 & 0.15625 \\ 0.3125 & 0.9375 & 0.54688 \\ 0.15625 & 0.54688 & 1.1719 \end{bmatrix}.$$

The initial conditions for the systems are $x(0) = [0.1, 0.5, 1.0]^T$. The estimator initial conditions are $\hat{x}(0) = [1, 0, 0]^T$. The initial parameter estimate is $\hat{\theta}(0) = 0.6$ with an uncertainty radius of $z_{\theta 0} = 0.5$. The initial uncertainty radius $z_{\eta 0}$ is set to 4. The upper bound on the uncertainty is $\bar{\omega} = 0.005$.

Figure 1 shows the performance of the estimation of θ_1. The parameter is shown to converge to the true value. A sudden change is introduced in the parameter value from 1 to 3 at t=50000. Figure 2 shows the upper bound on the observe error $\|He\|$ along with the upper bound as given in (39). As expected, inequality (39) accurately detect to the new conditions at time t=50000. As a result, the set update algorithms are reset starting at the current condition but with $z_\theta = 4$ and $z_\eta = 4$.

The radius of the parameter estimation error is compared to the norm of the parameter estimation error $\|\tilde{\theta}\|$ in Figure 3. The results confirm that $z_\theta \geq \|\tilde{\theta}\|$, as expected. Figure 4 compares $\|\tilde{\eta}\|$ with the bound z_η. As expected, z_η provides an upper bound on the estimation error for η.

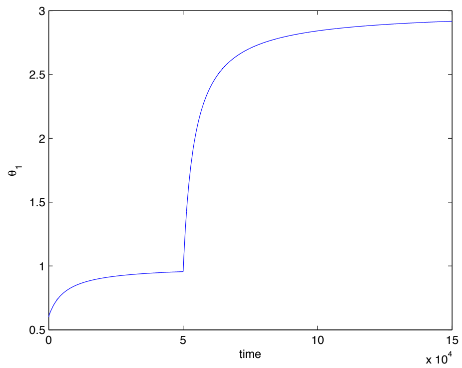

FIGURE 1. Time course plot of the parameter estimates.

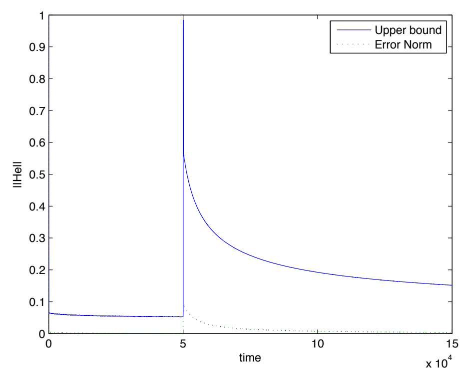

FIGURE 2. Comparison of the estimation error $\|e\|$ *and the upper bound based on* (39)

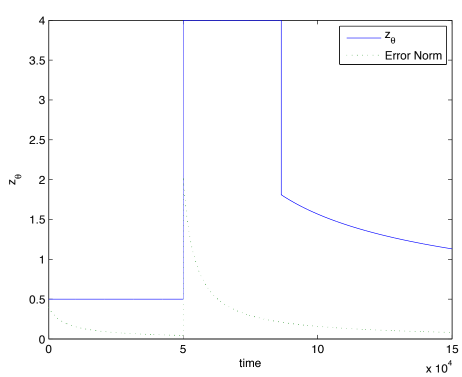

FIGURE 3. Radius of the uncertainty set z_θ *and true estimation error* $\|\tilde{\theta}\|$.

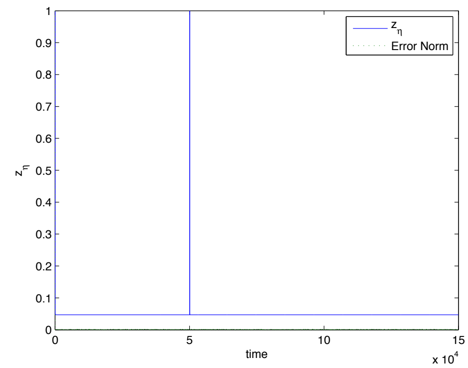

FIGURE 4. Radius of uncertainty z_η *and the true estimation error* $\|\tilde{\eta}\|$.

Conclusions

A set-based adaptive estimation technique is proposed for simultaneous state estimation and parameter identification of a class of continuous-time nonlinear systems subject to time-varying disturbances. The set-based adaptive identifier for parameters is used to estimate the parameters and along with an uncertainty that is guaranteed to contain the true value of the parameters. Simultaneously an auxiliary variable is used to estimate the unmeasured state variables. Sufficient conditions are given that guarantee the convergence of the adaptive observer. The proposed technique updates the estimates only when estimation improvement is guaranteed. The proposed uncertainty set update for parameter identification and state estimation, guarantees to contain the true values at all time instants. The method guarantees convergence of the parameter estimation error to zero and significantly determines the unknown state of the system with unknown bounded disturbance. The estimation and identification algorithms have been implemented to a simulation example to demonstrate its effectiveness.

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